

Banks, Liquidity Management and Monetary Policy*

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Abstract

We develop a new framework to study the implementation of monetary policy through the banking system. Banks make loans by issuing deposits. Loans are illiquid and, therefore, cannot be used to settle transfers of deposits. Instead, banks use central bank reserves for settlements but they may end short of reserves. This possibility induces a tradeoff between profiting from more loans against more liquidity risk exposure.

Monetary policy alters this tradeoff and consequently affects aggregate credit and interest rates. In turn, banks also react to shocks that alter the distribution of payments, induce bank equity losses, increase capital requirements, and cause contractions in the loans demand. We study how the effectiveness of monetary policy varies with these shocks. We calibrate our model to study, quantitatively, why have banks increased their liquidity holdings but not increased lending despite the policy efforts of recent years.

Keywords: Banks, Monetary Policy, Liquidity, Capital Requirements

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1 Introduction

The conduct of monetary policy around the world is changing. The past five years have witnessed banking systems that bore unprecedented financial losses and subsequent freezes in interbank markets. To protect themselves against a potential insolvency, banks cut back on their lending to the private sector. In response, the central banks of the US and Europe have reduced policy rates to almost zero, injected equity to the banking system and continuously purchased private paper in an open attempt to preserve financial stability and reinvigorate lending. However, in reaction to these unprecedented policy interventions, banks seem to have, for the most part, accumulated central bank reserves without renewing their lending activities as intended.¹ Why? Can central banks do more about this? These remain open questions.

Not surprisingly, the role of banks in the transmission of monetary policy has been at the center of policy debates. Unfortunately, there are few modern macroeconomic models that take into account that monetary policy is implemented through the banking system, as occurs in practice. Instead, most macroeconomic models assume that Central Banks control interest rates or the flow of credit directly and abstract from how the transmission of monetary policy may depend on the conditions of banks. This paper presents a model that contributes to filling this gap.

We use our model to answer a number of theoretical issues. What type of shocks can induce banks to hold more reserves and lend less? How does the transmission of monetary policy depend on the decisions of commercial banks? How does its strength vary with shocks to the banking system? In addition, we exploit the lessons derived from this theoretic framework to investigate, quantitatively, why are banks not lending despite all the policy efforts.

Our model is able to contrast different hypotheses that are informally discussed in policy and academic circles. Through the lens of the model, we evaluate the plausibility of the following hypothesis:

Hypothesis 1 - Bank Equity Losses: We study the hypothesis that the lack of lending responds to an optimal behavior by banks given the substantial equity losses suffered in 2008.

Hypothesis 2 - Capital Requirements: We analyze if the expected path of capital requirements are leading banks to hold more reserves and simultaneously lend less.

Hypothesis 3 - Increased Precautionary Holdings of Reserves: We also investigate if banks hold more reserves because they faced more uncertainty about potential costs of accessing the interbank market.

Hypothesis 4 - Interest on Excess Reserves: Banks are holding reserves and lending less because the excess reserves are earning an interest. This has lead to a substitution away from loans towards reserve holdings.

Hypothesis 5 - Weak Demand: Finally, we study if banks behave as if they face a weaker

¹As is well known, the Bank of Japan had been facing similar issues since the early nineties.

effective demand for loans. This hypothesis encompasses a direct shock to the demand for credit or a lack of borrowers that meet credit standards which leads to a weaker *effective* demand for loans.

We calibrate our model and fit it with shocks associated with each hypothesis. We use the its predictions to uncover which shocks are consistent with less lending in times when reserves have increased by several multiples. Our model suggests that a combination of shocks best fits the data. Overall, the model favors an early increase in disruptions in the interbank market followed by a substantial contraction in loan demand.

The Mechanism. The building block of our model is a liquidity management problem. Liquidity management is recognized as one of the fundamental problems in banking and can be explained as follows. When a bank grants a loan, it simultaneously creates demand deposits—or credit lines. These deposits can be used by the borrower to perform transactions at any time. Granting a loan is profitable because a higher interest is charged on the loan than what is paid on deposits. However, more lending relative to a given amount of central bank reserves increases a bank’s liquidity risks. When deposits are transferred out of a bank, that bank must transfer reserves to other banks in order to settle transactions. Central bank reserves are critical to clear settlements because, as occurs in practice, loans cannot be sold immediately. Thus, the lower the reserve holdings of a bank, the more likely it is to be short of reserves in the future. This is a source of risk because the bank must incur in expensive borrowing from other banks—or the Central Bank’s discount window—if it ends short of reserves. This friction—the liquidity mismatch—induces a trade-off between a profiting from lending against additional liquidity risks. Bank lending reacts to monetary policy because its policy instruments alter this tradeoff.

We introduce this liquidity management problem and an interbank market into a tractable dynamic general-equilibrium model with rational profit-maximizing banks. Bank liquidity management is captured through a portfolio problem with non-linear returns that depend on the bank’s reserve position. We use this to study the effects of shocks to banks affect that their aggregate lending and reserve holdings.

Implementing Monetary Policy. In the model, a Central Bank is equipped with various tools. A first set of instruments are discount rates and interests on reserves which influence the costs of being short of reserves. A second set are reserve requirements, open-market operations and direct lending to banks. This latter set of instruments, alters the effective aggregate amount of reserves in the system. All of these instruments carry real effects by tilting the liquidity management tradeoff. Macroeconomic effects result from their indirect effect on aggregate lending—and interest rates. However, as much as a Central Bank can influence bank decisions, the shocks associated with each hypothesis limit the power of monetary policy.

Testable Implications. The model delivers a rich set of descriptions. For individual banks, it explains the behavior of their reserve ratio, their leverage ratio and their dividend policies. Aggregating across banks provides descriptions for aggregate lending, interbank lending volumes

and excess reserves. In general equilibrium, this yields predictions for interbank and non-interbank borrowing and lending rates. The model also describes other financial indicators for banks. For example, the return on loans, the return on equity, dividend ratios and book and market equity values. Moreover, the model also yields predictions for the evolution of the financial sector’s equity. At the macroeconomic level, the model also explains the evolution of an endogenous money multiplier. We use this rich descriptions to identify the shocks associated with hypotheses 1-4. This allows us to shed light on which of the four hypotheses best fits the patterns we have seen since the 2008-2009 financial crisis.

Organization. The paper is organized as follows. The following section discusses where the model fits in the literature. Section 2, presents the model and some theoretical results. Section 4 presents a calibration exercise. We study the steady state and policy functions under that calibration in Section 6. We study transitional dynamics after an unexpected shocks associated with each hypothesis in section 7. Finally, in Section 8 we evaluate and discuss the plausibility each hypotheses.

1.1 Related Literature

There is a tradition in macroeconomics that dates back at to least [Bagehot \(1873\)](#) which stresses the importance of analyzing monetary policy in conjunction with banks. A classic mechanical framework to study policy with a full description of households, firms and banks is [Gurley and Shaw \(1964\)](#). With few exceptions, modeling banks was abandoned from macroeconomics for many years. Until the Great Recession, the macroeconomic effects of monetary policy and its implementation through banks were analyzed independently.²

In the aftermath of the crisis, however, there have been numerous calls for constructing models with an explicit role for banks.³ Some early steps have been taken by [Gertler and Karadi \(2009\)](#) and [Curdia and Woodford \(2009\)](#). In those models, shocks to bank equity —coupled with leverage constraints— propagate because they interrupt financial intermediation and increase spreads. The focus of those papers to explain the effects of policies that recapitalize banks. In contrast, policy effects in our model arise from differences in the liquidity of assets. This relates our model to classic models of bank liquidity management and monetary policy.⁴ Our contribution to bring the classic insights from the liquidity-management literature into a modern general-equilibrium

²This was a natural simplification by the literature. In the US, the behaviour of banks did not seem to matter for monetary policy. In fact, the banking industry was among the most stable industries in terms of returns and the pass-through from policy tools to aggregate conditions had little variability.

³See for example [Woodford \(2010\)](#) and [Mishkin \(2011\)](#).

⁴Classic papers that study static liquidity management —also called reserve management— by individual banks are [Poole \(1968\)](#) and [Frost \(1971\)](#). There are many modern textbooks for practitioners that deal with liquidity management. For example, [Saunders and Cornett \(2010\)](#) and [Duttweiler \(2009\)](#) provide managerial and operations research perspectives. Many modern banking papers have focused on bank runs. See for example [Diamond and Dybvig \(1983\)](#), [Allen and Gale \(1998\)](#), [Ennis and Keister \(2009\)](#), or [Holmström and Tirole \(1998\)](#). [Gertler and Kiyotaki \(2013\)](#) is a recent paper that incorporates bank runs into a dynamic macroeconomic model.

dynamic model that can be used for the analysis of policy and banking crises.

We share common elements with recent work by [Brunnermeier and Sannikov \(2012\)](#). [Brunnermeier and Sannikov \(2012\)](#) also introduce inside and outside money into a dynamic macro model. Their focus is on the real effects of monetary policy through the redistributive effects of inflation. In that sense, their model is closer to [Gertler and Karadi \(2009\)](#) and [Curdia and Woodford \(2009\)](#) because the distribution of wealth affects the extent of financial frictions. In our setup, outside money does not circulate outside the banking sector. Instead, central bank reserves only serve as an instruments to settle payments among banks.

The use of reserves for precautionary motives also places the model close to [Stein \(2012\)](#) and [Stein et al. \(2013\)](#). Those papers study the effects of an increase in the supply of reserves given an exogenous demand for short-term liquid assets. [Williamson \(2012\)](#) studies an environment where assets of different maturity have different properties as mediums of exchange. Our paper builds on earlier insights from two papers in the money-search literature. In particular, [Cavalcanti et al. \(1999\)](#) provide a theoretical foundation to our setup because reserves there emerge as disciplining device to sustain credit creation under moral-hazard and guarantee the circulation of deposits. In turn, we model an interbank market building on earlier work by [Afonso and Lagos \(2012\)](#). That paper models the Fed-funds market as an over-the-counter market where illiquidity costs arise endogenously. Our market for reserves is a simplified version of that model. On the technical side, our paper relates to [Corbae and D’Erasmus \(2013\)](#) who study a dynamic model of the banking industry with heterogeneity.

2 The Model

The description of the model begins with a partial-equilibrium dynamic model of banks. The goal is to derive the supply of loans and the demand for reserves given an exogenous demand for loans, central bank policies and aggregate shocks. In the appendix, we formally derive a demand for loans and deposits from to close the model.

2.1 Environment

Time is discrete, is indexed by t and there is an infinite horizon. Each period is divided into two stages: a lending stage (l) and a balancing stage (b). The economy is populated by a continuum of competitive banks whose identity is denoted by z . Banks face an exogenous demand for loans and a vector of shocks that we describe later. There is an exogenous deterministic monetary policy chosen by the monetary authority which we refer to as the Fed. There are three types of assets, deposits, loans and central bank reserves. Deposits and loans are denominated in real terms. Reserves are denominated in nominal terms. Deposits play the role of a numeraire.

Banks. A bank’s preferences over real dividend streams $\{DIV_t\}_{t \geq 0}$ are evaluated via an

expected utility criterion:

$$\mathbb{E}_0 \sum_{t \geq 0} \beta^t U(DIV_t)$$

where $U(DIV) \equiv \frac{DIV^{1-\gamma}}{1-\gamma}$ and DIV_t is the banker's consumption at date t .⁵ Banks hold a portfolio of loans, B_t , and central bank reserves, C_t , as part of their assets. Demand deposits, D_t , are their only form of liabilities. These holdings are the individual state variables of a bank.

Loans. Banks make loans during the lending stage. The flow of new loan issuances is I_t . These loans constitute a promise to repay the bank $I_t(1-\delta)\delta^n$ in period $t+1+n$ for all $n \geq 0$, in units of numeraire. Thus, loans promise a geometrically decaying stream of payments as in the Leland-Toft model (see [Leland and Toft, 1996](#)). We denote by B_t the stock of loans held by a bank at time t . Given the structure of payments, the stock of loans has a recursive representation:

$$B_{t+1} = \delta B_t + I_t.$$

When banks give a loan, they provide the borrower a demand deposits account which amount to $q_t^l I_t$, where q_t is the price of the loan. Banks take q_t as given. Consequently, the bank's immediate accounting profits are $(1 - q_t^l) I_t$.

A key feature of our model is that bank loans are illiquid—they cannot be sold or bought—during the balancing stage.⁶ The lack of a liquid market for loans in the balancing stage can be rationalized by several market frictions. For example, loans may be illiquid assets if banks specialize in particular customers or if they face agency frictions.⁷

Demand Deposits. Deposits earn a real gross interest rate $R^D = (1 + r^d)$. Behind the scenes, banks enable transactions between third parties. When they obtain a loan, borrowers receive deposits. This means that banks make loans—a liability for the borrower—by issuing their own liabilities—an asset ultimately held by a third party. This swap of liabilities enables borrowers to purchase goods because deposits are effective mediums of exchange. After the transaction is made, the holder of those deposits, may, in turn transfer those funds again to the accounts of others, make payments and so on.

A second key feature of the environment is that deposits are callable on demand. In the balancing stage, banks are subject to random withdrawals of deposits given by $\omega_t D_t$, where $\omega_t \sim F_t(\cdot)$

⁵Introducing curvature into the objective function is important. This assumption generates smooth dividends and slow-moving bank equity, as observed empirically. Similar preferences are often found in the corporate finance literature. One way to rationalize these preferences is through undiversified investors that hold bank equity. Alternatively, agency frictions may induce equity adjustment costs.

⁶The assumption that loans can be sold during the lending stage allows us to reduce the state space. In particular, it is not necessary to keep track of the composition but only the size of bank balance sheets thanks to this assumption. Dispensing this assumption would require us to keep track of a non-degenerate cross-sectional distribution for reserves, deposits and loans.

⁷[Diamond \(1984\)](#) and [Williamson \(1987\)](#) introduce specialized monitoring technologies. [Holmstrom and Tirole \(1997\)](#) build a model where bankers must hold a stake on the loans because of moral hazard. Finally, [Bolton and Freixas \(2009\)](#) introduce a differentiated role for different bank liabilities following from asymmetric information.

with support in $(-\infty, 1]$. Here, F_t is the time-varying cumulative distribution for withdrawals. For simplicity, we assume F_t is common to all banks.⁸ When ω_t is positive (negative), the bank loses (receives) deposits. The shock ω_t captures the idea above that deposits are constantly circulating when payments are executed. The complexity of these transactions is approximated by the random process of ω_t . For simplicity, we assume that deposits do not leave the banking sector:

Assumption 1 (Deposit Conservation). *Deposits remain within the banking system: $\int_{-\infty}^1 \omega_t dF_t(\omega) = 0, \forall t$.*

This assumption implies that there are no withdrawals of reserves outside the banking system.⁹

When deposits are transferred across banks, the receptor bank absorbs a liability issued by another bank. Therefore, this transaction needs to be settled with the transfer of an asset. Since bank loans are illiquid, deposit transfers are settled with reserves. Thus, the illiquidity of loans induces a demand for reserves. The stock of deposits held by a bank is altered as borrowers repay their loans over time or as banks issue deposits to buy loans or reserves from other banks during the lending stage.

Reserves. Reserves are special assets. They are issued by the Fed and used by banks to settle transactions. Banks can buy or sell reserves during the lending stage. However, during the balancing stage, they can only borrow or lend reserves in the interbank market —details below. We denote by p_t be the price of reserves in terms of deposits. This term is also the inverse of the price level because deposits are in real terms.

By law, banks must hold a minimum amount of reserves within the balancing stage. In particular, the law states that $p_t C_t \geq \rho D_t(1 - \omega_t)/R^D$, where $\rho \in [0, 1]$ is a reserve requirement chosen by the Fed.¹⁰ The case $\rho = 0$ requires banks to finish with a positive balance of reserves —banks cannot issue these liabilities. Given the reserve requirement, if ω_t is large, reserves may be insufficient to settle the outflow of deposits. In turn, banks that receive a large unexpected inflow will hold reserves in excess of the requirement.

To meet reserve requirements or allocate reserves in excess, banks can lend and borrow from each other or from the Fed. These trades constitute the interbank market. As part of its toolbox, the Fed chooses two policy rates: a lending rate, r_t^{DW} , and a borrowing rate, r_t^{ER} . The borrowing rate —the interest on excess reserves —is the interest paid by the Fed to banks who deposit excess reserves at the Fed. The lending rate —or discount window rate— is the rate at which the Fed lends reserves to banks in deficit. These rates satisfy $r_t^{DW} \geq r_t^{ER}$ and are paid within the period

⁸We could assume that F is a function of the bank's liquidity or leverage ratio. This would add complexity to the bank's decisions but would not break any aggregation result. This tractability is lost if F_t is a function of the bank's size.

⁹This assumption can be relaxed without problem to allow for a demand for currency or system-wide bank-runs at an extreme.

¹⁰Some operating frameworks compute reserve balances over a maintenance period. Bank choices in our model would correspond to averages over that period.

with deposits.¹¹ Banks have the option to trade with the Fed or to trade with other banks.

Interbank Market. We assume that the interbank market for reserves is a directed over-the-counter (OTC) market.¹² This interbank market works in the following way. After the realization of withdrawal shocks, banks end with either positive or negative balances relative to their reserve requirements. A bank that wishes to lend a dollar in excess can place a lending order. A bank that needs to borrow a dollar to patch its deficit can place a borrowing order. Thus, orders are placed on a per-unit basis as in [Atkeson et al. \(2012\)](#). Orders are directed to either the borrowing or lending sides of the market. After orders are directed to either side, a dollar in excess is randomly matched with a dollar in deficit. Once a match is realized, the lending bank can transfer the unit overnight. Banks use Nash bargaining to split the surplus of the dollar transfer.

In the bargaining problem that emerges, the outside option for the lending bank is to deposit the dollar at the Fed earning r^{ER} . For the bank in deficit, the outside option is the discount window rate r^{DW} . Because the principle of the loan—the dollar itself—is returned by the end of the period, banks bargain only about the net rate. We call this net rate the Fed funds rate, r^{FF} .

The bargaining problem for a match is:

Problem 1 (Interbank-market bargaining problem)

$$\max_{r^{FF}} (m_l r_t^{DW} - m_l r^{FF})^\xi (m_b r^{FF} - m_b r_t^{ER})^{1-\xi}.$$

In the objective function, m_l is the marginal utility of the bank lending reserves and m_b the corresponding term for banks borrowing. The first order condition of this problem is:

$$\frac{(r^{FF} - r_t^{ER})}{((1 + r_t^{DW}) - (1 + r^{FF}))} = \frac{(1 - \xi)}{\xi}.$$

This condition yields an implicit solution for r^{FF} . Since $(1 - \xi) / (\xi)$ is positive, it is clear that r^{FF} will fall within the Fed's corridor of interest rates, $[r_t^{ER}, r_t^{DW}]$.

Not every order is necessarily matched. Instead, the probabilities that a lending order will meet borrowing order depends on the relative masses on either side of the market. In particular, γ^- is the probability that a deficit dollar is matched with a surplus dollar. We denote by M^+ the mass of lending orders and by M^- the mass of borrowing orders. The probability that a borrowing order finds a lending order is given by $\gamma^- = \min(1, M^+/M^-)$. Conversely, the probability that a lending order finds a borrowing order is $\gamma^+ = \min(1, M^-/M^+)$. The probabilities will affect the average cost of being short or long of reserves.¹³

¹¹This determines what in practice is known as the corridor system. In practice, there is an additional wedge between these two rates associated with the stigma from borrowing from the Fed.

¹²The features of the interbank market are borrowed from work by [Afonso and Lagos \(2012\)](#), [Ashcraft and Duffie \(2013\)](#) and [Duffie \(2012\)](#).

¹³There are a few implicit conventions here. First, if an order does not find a match, the bank does not lose the opportunity to lend (borrow) from (to) the Fed. Second, a bank cannot place orders beyond its reserve needs or

Bank Equity and Payouts. The market value of equity is defined as $E_t = q_t B_t + p_t C_t - D_t$. This term evolves depending on prices and the realization of bank profits. Finally, dividend payouts occur during the lending stage.

2.2 Timing, Laws of Motion and Bank Problems

This section shows expresses the model recursively. Thus, we drop time subscripts from now on. We adopt the following notation: If Z is a variable at the beginning of the period, \tilde{Z} is its value by the end of the lending stage and the beginning of the balancing stage. Similarly, Z' is its value by the end of the balancing stage and the beginning of the following period. The aggregate state includes: all policy decisions by the Fed, the distribution of withdrawal shocks, F , and the demand for loans—to be specified below. This aggregate state is summarized in the vector X . We denote by V^l and V^b as the bank's value function during the lending and balancing stages.

Lending Stage. Banks enter the lending stage with reserves, C , loans, B , and deposits, D . The bank chooses dividends, DIV , loan issuances, I , and purchases of reserves, φ .¹⁴ The evolution of deposits follows:

$$\frac{\tilde{D}}{R^D} = D + qI + DIV + \varphi p - B(1 - \delta).$$

Several actions affect this evolution. First, deposits increase when the bank credits qI deposits in the accounts of borrowers—or whomever they trade with. Second, banks pay dividends to shareholders with deposits. Third, the bank issues $p\varphi$ deposits to buy φ reserves. Finally, deposits fall by $B(1 - \delta)$ because loans are amortized with deposits.

At the end of the lending stage reserves are the sum of the previous stock plus purchases of reserves, $\tilde{C} = C + \varphi$. Loans evolve according to $\tilde{B} = \delta B + I$. Banks choose $\{I, DIV, \varphi\}$ subject to these laws of motion and a capital requirement constraint. The capital requirement constraint imposes an upper bound, κ , on the stock of deposits relative to equity—marked-to-market. The bank's problem in the lending stage is:¹⁵

excess—without this restriction, banks could place higher orders to increase their probabilities of matching. Finally, interests are paid with deposits—this is just a convention since all assets are liquid during the lending stage.

¹⁴The purchase of reserves φ occurs during the lending stage. Thus, this is a different flow than the flow that follows from loans in the interbank market which occurs during the balancing stage.

¹⁵On the technical side, the capital requirement constraint bounds the bank's problem and prevents a Ponzi scheme. It is important to note that if the bank arrives to a node with negative equity, the problem is not well defined. However, when choosing its policies, the bank will make decisions that guarantees that it does not run out of equity. Implicitly, it is assumed that if the bank violates any constraint, it goes bankrupt.

Problem 2 *In the lending stage, banks solve:*

$$\begin{aligned}
V^l(C, B, D; X) &= \max_{I, DIV, \varphi} U(DIV) + \mathbb{E} \left[V^b(\tilde{C}, \tilde{B}, \tilde{D}; \tilde{X}) \right] \\
\frac{\tilde{D}}{R^D} &= D + qI + DIV + p\varphi - B(1 - \delta) \\
\tilde{C} &= C + \varphi \\
\tilde{B} &= \delta B + I \\
\frac{\tilde{D}}{R^D} &\leq \kappa \left(q\tilde{B} + p\tilde{C} - \frac{\tilde{D}}{R^D} \right); \tilde{B}, \tilde{C}, \tilde{D} \geq 0.
\end{aligned}$$

Balancing Stage. During the balancing stage, banks place orders in the interbank market or at the Fed. Loans remain unchanged. However, the withdrawal $\omega\tilde{D}$ shifts the holdings of deposits and reserves. Let x be the reserve deficit. Given that withdrawals are settled with reserves, this deficit is:

$$x = \underbrace{\rho \left(\frac{\tilde{D} - \omega\tilde{D}}{R^D} \right)}_{\text{End-of-Stage Deposits}} - \underbrace{\left(\tilde{C}p - \frac{\omega\tilde{D}}{R^D} \right)}_{\text{End-of-Stage Reserves}}.$$

Given the structure of the OTC market described earlier, a bank with reserve surplus obtains a return of r^{FF} if it lends a unit of reserves in the interbank market and r^{ER} if it lends to the Fed. Since for any Nash-bargaining parameter $r^{FF} > r^{ER}$, banks always attempt to lend first in the interbank market. Thus, they place lending orders for every dollar in excess. In equilibrium, only a fraction γ^+ of those orders are matched and earn a return of r^{FF} . The rest earns the Fed's borrowing rate r^{ER} . Thus, the average return on excess reserves is:

$$\chi_l = \gamma^+ r^{FF} + (1 - \gamma^+) r_t^{ER}$$

Analogously, banks in deficit try to first borrow from other banks before borrowing from the Fed because $r^{FF} < r_t^{DW}$. The analogous cost of reserve deficits is:

$$\chi_b = \gamma^- r^{FF} + (1 - \gamma^-) r_t^{DW}.$$

The difference between χ_l and χ_b is an endogenous wedge between the marginal value of excess reserves and costs of reserve deficits. The simple rule that characterizes orders in the interbank market problem, yields a value function for the bank during the balancing stage:

Problem 3 *The value of the Bank's problem during the balancing stage is:*

$$\begin{aligned}
V^b(\tilde{C}, \tilde{B}, \tilde{D}; \tilde{X}) &= \beta \mathbb{E} \left[V^l(C', B', D'; X') | \tilde{X} \right] \\
D' &= \tilde{D}(1 - \omega) + \chi(x) \\
B' &= \tilde{B} \\
x &= \rho \left(\frac{\tilde{D} - \omega \tilde{D}}{R^D} \right) - \left(\tilde{C}p - \frac{\omega \tilde{D}}{R^D} \right) \\
C' &= \tilde{C} - \frac{\omega \tilde{D}}{p}.
\end{aligned}$$

Here χ represents an illiquidity cost, the return/cost of excess/deficit of reserves:

$$\chi(x) = \begin{cases} \chi_l x & \text{if } x \leq 0 \\ \chi_b x & \text{if } x > 0 \end{cases}$$

We can collapse the problem of a bank for the entire period through a single Bellman equation that by substituting V^b into V^l :

Problem 4 *The bank's problem during the lending stage is:*

$$\begin{aligned}
V^l(C, B, D, X) &= \max_{\{I, DIV, \tilde{C}, \tilde{D}\} \in \mathbb{R}_+^4} U(DIV) \dots \\
&\quad + \beta \mathbb{E} \left[V^l \left(\tilde{C} - \frac{\omega' \tilde{D}}{p}, \tilde{B}, \tilde{D}(1 - \omega') + \chi \left(\frac{(\rho + \omega'(1 - \rho)) \tilde{D}}{R^D} - \tilde{C}p \right) \right); X' | X \right] \\
\frac{\tilde{D}}{R^D} &= D + qI + DIV_t + p\varphi - B(1 - \delta) \tag{1} \\
\tilde{B} &= \delta B + I \tag{2} \\
\tilde{C} &= \varphi + C \tag{3} \\
\frac{\tilde{D}}{R^D} &\leq \kappa \left(\tilde{B}q + \tilde{C}p - \frac{\tilde{D}}{R^D} \right).
\end{aligned}$$

The following section provides a characterization of this problem.

2.3 Characterization of Bank Problem

The recursive problem of banks can be characterized through a single state variable, the banks' equity value after loan amortizations, $E \equiv pC + B(1 - \delta + \delta q) - D$. To show this, we clear out I and φ from the laws of motion of loans and reserves, equations (2) and (3), and substitute out I and φ into the law of motion for deposits, equation (1). After substitutions, the evolution of

deposits takes the form of a budget constraint:

$$q\tilde{B} + \tilde{C}p + DIV - \tilde{D} = E.$$

In this budget constraint E is the value of the bank's available resources, which is predetermined. We use an updating rule for E that depends on the bank's current decisions to express the bank's value function through a single-state variable:

Proposition 1 (Single-state Representation)

$$\begin{aligned} V(E) &= \max_{\tilde{C}, \tilde{B}, \tilde{D}, DIV \in \mathbb{R}_+^4} U(DIV) + \beta \mathbb{E}[V(E')|X] \\ E &= q\tilde{B} + p\tilde{C} + DIV - \frac{\tilde{D}}{R^D} \\ E' &= (q'\delta + 1 - \delta)\tilde{B} + p'\tilde{C} - \tilde{D} - \chi \left(\frac{(\rho + \omega'(1 - \rho))\tilde{D}}{R^D} - \tilde{C}p \right) \\ \frac{\tilde{D}}{R^D} &\leq \kappa \left(\tilde{B}q + \tilde{C}p - \frac{\tilde{D}}{R^D} \right). \end{aligned} \tag{4}$$

This problem resembles a standard consumption-savings decision problem subject to a leverage constraint. Dividends play the role of consumption, the bank's savings are allocated into loans, \tilde{B} , and reserves, \tilde{C} , and it can lever its position issuing deposits \tilde{D} .¹⁶ Its choice is subject to a capital requirement constraint—the leverage constraint. The budget constraint is linear in E and the objective is homothetic. Thus, by the results in [Alvarez and Stokey \(1998\)](#), the solution to this problem exists, is unique, and policy functions are linear in equity. Formally,

Proposition 2 (Homogeneity— γ) *The value function $V(E; X)$ satisfies*

$$V(E; X) = v(X) E^{1-\gamma}$$

where $v(\cdot)$ satisfies

$$v(X) = \max_{\tilde{c}, \tilde{b}, \tilde{d}, div \in \mathbb{R}_+^4} U(div) + \beta \mathbb{E}[v(X')|X] \mathbb{E}_{\omega'}(e')^{1-\gamma} \tag{5}$$

¹⁶From here on, we use the terms cash and reserves interchangeably. We acknowledge that in the context of non-depository institutions, cash may mean holdings of deposits.

subject to

$$\begin{aligned}
1 &= q\tilde{b} + p\tilde{c} + \text{div} - \frac{\tilde{d}}{R^D} \\
e' &= (q'\delta + (1-\delta))\tilde{b} + p'\tilde{c} - \tilde{d} - \chi((\rho + \omega'(1-\rho)) \frac{\tilde{d}}{R^D} - p\tilde{c}) \\
\frac{\tilde{d}}{R^D} &\leq \kappa \left(q\tilde{b} + \tilde{c}p - \frac{\tilde{d}}{R^D} \right)
\end{aligned}$$

Moreover, the policy functions in (4) satisfy $X = xE$. In the expression above, $\mathbb{E}_{\omega'}$ is the expectation under F .

According to this proposition, the policy functions in (4) can be recovered from (5) by scaling them by equity, i.e., if c^* is the solution to (5), we have that $C = Ec^*$, and the same applies for the rest of the policy functions. An important implication is that two banks with different equity are scaled versions of a bank with one unit of equity. This also implies that the distribution of equity is not a state variable, but rather only the aggregate value of equity. Moreover, although there is no invariant distribution for bank equity —the variance of distribution grows over time—, the model yields predictions about the cross-sectional dispersion growth.

An additional useful property of the bank's problem is that it satisfies portfolio separation. In particular, the choice of dividends can be analyzed independently —through a consumption savings problem with a single asset— from the portfolio choices between deposits, reserves and loans. We use the principle of optimality to break the Bellman equation (5) into two components.

Proposition 3 (Separation) *The value function $v(\cdot)$ defined in (5) solves:*

$$v(X) = \max_{\text{div} \in \mathbb{R}_+} U(\text{div}) + \beta \mathbb{E}[v(X') | X] \Omega(X)^{1-\gamma} (1 - \text{div})^{1-\gamma}. \quad (6)$$

Here $\Omega(X)$ is the value of the certainty-equivalent portfolio value of the bank. $\Omega(X)$ is the outcome of the following liquidity-management portfolio problem:

$$\begin{aligned}
\Omega(X) &\equiv \max_{\{w_b, w_c, w_d\} \in \mathbb{R}_+^3} \left\{ \mathbb{E}_{\omega'} [R_X^B w_b + R_X^C w_c - w_d R_X^D - R_X^\chi(w_d, w_c)]^{1-\gamma} \right\}^{\frac{1}{1-\gamma}} \\
w_b + w_c - w_d &= 1 \\
w_d &\leq \kappa(w_b + w_c - w_d)
\end{aligned} \quad (7)$$

with $R_X^B \equiv \frac{q'\delta + (1-\delta)}{q}$, $R_X^C \equiv \frac{p'}{p}$, $R_X^\chi \equiv \chi((\rho + \omega'(1-\rho)) w_d - w_c)$.

Once we solve the policy functions of this portfolio problem, we can reverse the solution for $\tilde{c}, \tilde{b}, \tilde{d}$ that solve (5) via the following formulas: $\tilde{b} = (1 - \text{div}) w_b / q$, $\tilde{c} = (1 - \text{div}) w_c / p$ and $\tilde{d} = (1 - \text{div}) w_d R^D$.

The maximization problem that determines $\Omega(X)$ consists of choosing portfolio shares among assets of different risk, liquidity, and return. This problem is a liquidity-management portfolio problem with the objective of maximizing the certainty equivalent return on equity: $R^E(\omega'; w_b, w_d, w_c) \equiv R^B w_b + R^C w_c - R^D w_d - R^X(w_d, w_c, \omega')$. This portfolio problem is not a standard portfolio problem as it features non-linear returns that result from the joint determination of the return on reserves and deposits —given the illiquidity of loans and the reserve requirements. The return on loans is the sum of the coupon payment plus resale price: $R^B \equiv (\delta q + (1 - \delta)) / q$. The return on reserves and deposit components of the portfolios is determined jointly and depends on the withdrawal shock ω . These returns can be separated into an independent return and a joint return component that follows from the characteristics of the interbank market. The independent return on reserves is given by the deflation rate $R^C \equiv p' / p$.¹⁷ The independent return of deposits is the interest on deposits, R_X^D . The joint return component of reserves and deposits is the potential cost —or benefit— of running out of reserves. This illiquidity cost is given by:

$$R_X^X(w_d, w_c, \omega') \equiv \chi((\rho + (1 - \rho)\omega')w_d - w_c). \quad (8)$$

The risk and return of each assets varies with aggregate variables X . Thus, the solutions to the liquidity-management portfolio problem are time varying —outside steady state. The solution for the dividend rate and marginal values of bank equity satisfy a system of equations:

Proposition 4 (Solution for dividends and bank value) *Given the solution to the portfolio problem γ the dividend ratio and value of bank equity are given by:*

$$\text{div}(X) = \frac{1}{1 + [\beta(1 - \gamma)\mathbb{E}[v(X')|X]\Omega^*(X)^{1-\gamma}]^{1/\gamma}}$$

and

$$v(X) = \frac{1}{1 - \gamma} \left[1 + (\beta(1 - \gamma)\Omega^*(X)^{1-\gamma} \mathbb{E}[v(X')|X])^{\frac{1}{\gamma}} \right]^\gamma.$$

The policy functions of banks determine the loans supply and demand for reserves. This concludes the partial equilibrium analysis of the bank's portfolio decisions. We now describe the demand for loans and the actions of the Fed.

2.4 Loan Demand

We consider a downward sloping demand for loans with respect to the loan rate, i.e. increasing on the price. In particular, we consider a constant elasticity (CES) demand function:

$$q_t = \Theta_t (I_t^D)^\epsilon, \epsilon > 0, \Theta_t > 0. \quad (9)$$

¹⁷For most of the analysis we set $p' = p$, so that reserves yield a return equal to zero.

where ϵ is the inverse of the semi-elasticity of credit demand with respect to the price and Θ_t are possible demand shifters. In the quantitative analysis, consider shocks to Θ_t to evaluate the extent of shocks to aggregate demand for loans —hypothesis 4. We analyze in section ?? a microfoundation for this demand of loans, based on a working capital constraint.

2.5 The Fed's Balance Sheet and its Operations

This section describes the Fed's balance sheet and how the Fed implements monetary policy. The Fed's balance sheet is analogous to that of commercial banks with an important exception: the Fed doesn't issue demand deposits as liabilities, but issues reserves instead. As part of its assets, the Fed holds commercial bank deposits, D_t^{Fed} , and private sector loans, B_t^{Fed} . As liabilities, the Fed issues M_t^0 reserves —high power money. The Fed's assets and liabilities satisfy the following laws of motion:

$$\begin{aligned} M_{t+1}^0 &= M_t^0 + \varphi_t^{Fed} \\ \frac{D_{t+1}^{Fed}}{R^D} &= D_t^{Fed} + p_t \varphi_t^{Fed} + (1 - \delta) B_t^{Fed} - q_t I_t^{Fed} + \chi_t^{Fed} - T_t \\ B_{t+1}^{Fed} &= \delta B_t^{Fed} + I_t^{Fed}. \end{aligned}$$

The laws of motion for these state variables are very similar to the laws of motion for banks. Here, φ_t^{Fed} represents the Fed's purchase of deposits by issuing reserves to commercial banks. Its deposits are affected by the purchase or sale of loans, I_t^{Fed} , and the coupon payments of previous loans, $(1 - \delta) B_t^{Fed}$. In addition, the Fed's deposits vary with, T_t , the transfers to or from the fiscal authority —the analogue of dividends. Finally, χ_t^{Fed} represents the Fed's income revenue that stems from its participation in the Fed funds market:

$$\chi_t^{Fed} = \underbrace{r_t^{DW} (1 - \gamma^-) M^-}_{\text{Earnings from Discount Loans}} - \underbrace{r_t^{ER} (1 - \gamma^+) M^+}_{\text{Losses from Interest Payments on Excess Reserves}}.$$

The Fed's balance sheet constraint is obtained by combining the laws of motion for reserves, loans and deposits:

$$p_t (M_{t+1}^0 - M_t^0) + (1 - \delta) B_t^{Fed} + \chi_t^{Fed} = D_{t+1}^{Fed}/R^D - D_t^{Fed} + q_t (B_{t+1}^{Fed} - \delta B_t^{Fed}) + T_t. \quad (10)$$

Expressed in real terms, the Fed's balance sheet is the following identity:

$$\underbrace{E_t^{Fed}}_{\text{Fed equity}} = \underbrace{D_t^{Fed} + ((1 - \delta) + q_t \delta) B_t^{Fed}}_{\text{Assets}} - \underbrace{p_t M_t^0}_{\text{Liabilities}}.$$

where E_t^{Fed} is the Fed's equity. The Fed has a monopoly on the supply of reserves, M_t^0 and alters this quantity through several operations.

Unconventional Open-Market Operations. Since there are no government bonds, only unconventional monetary operations are available.¹⁸ An unconventional OMO involves the purchase of loans and the issuance of reserves. This operation does not affect the stock of commercial bank deposits held by the Fed. To keep the amount of deposits constant, the Fed issues M^0 buying deposits from banks but then sells those deposits to purchase loans. Thus, an unconventional OMO satisfies:

$$p_t \Delta \varphi_t^{Fed} - q_t \Delta I_t^{Fed} = 0$$

where $\Delta \varphi_t^{Fed}$ and ΔI_t^{Fed} are the changes in deposits and loans corresponding to the OMO. Thus, the effect on the supply of reserves is $\Delta M_t^0 = \frac{q_t}{p_t} \Delta I_t^{Fed}$.

Open-Market Liquidity Facilities. Liquidity facilities are swaps of liabilities of the Fed for deposits in a way that keeps M_0 constant—a change in $\Delta \varphi_t^{Fed}$ without an offsetting ΔI_t^{Fed} .

Fed Profits and Transfers. In equilibrium, the Fed can return profits or losses. These operational results follow from the return on the Fed's loans and its profits/losses in the interbank market χ_t^{Fed} . We assume that the Fed transfers losses or profits immediately.

Fed Targets. For the analysis of the transitional dynamics we assume that the Fed chooses a target for the Fed funds rate r_t^{FF} —effectively setting r_t^{DW} and r_t^{ER} .

In our transition dynamics, experiments we consider a monetary policy regime where the Fed chooses the value of MO_t to maintain price stability: $p_t = p$.¹⁹

2.6 Market Clearing, Evolution of Bank Equity and Equilibrium

Bank Equity Evolution. Define by $\bar{E}_t \equiv \int_0^1 E_t(z) dz$, as the aggregate of equity in the banking sector. The equity of an individual bank evolves according to $E_{t+1}(z) = e_t(\omega) E_t(z)$. Here, $e_t(\omega)$ is the growth rate of bank equity of a bank with withdrawal shock ω . The measure of equity holdings at each bank is denoted by Γ_t . Since the model is scale invariant, we only need to keep track of the evolution of average equity, $\int_0^1 E_t(z) dz$, which by independence grows at rate $E_\omega[e_t]$.²⁰

Loans Market. Market clearing in the loans market requires us to equate the loans demand

¹⁸Incorporating Treasury Bills (T-bills) and conventional open market operations into our model is relatively straight forward. If T-bills are illiquid in the balancing stage, T-Bills and loans become perfect substitutes from a bank's perspective and the model becomes equivalent to our baseline model, with an additional market-clearing condition for T-bills. If T-Bills are perfectly liquid, we can show that banks that have a deficit in reserves sell first their holdings of Treasuries before accessing the interbank market. In the intermediate case where T-Bills are imperfect substitutes, the price of T-Bills would depend on the distribution of assets in the economy.

¹⁹It is straightforward to consider alternative monetary policy regimes, but the objective of price stability is both parsimonious and has empirical support.

²⁰A limiting distribution for Γ_t is not well-defined unless one adapts the process for equity growth.

I_t^D to the supply of new loans made by banks and the Fed. Hence, equilibrium must satisfy:

$$I_t^D \equiv (q_t/\Theta_t)^{\frac{1}{\epsilon}} = B_{t+1} - \delta B_t + B_{t+1}^{Fed} - \delta^T B_t^{Fed}. \quad (11)$$

Money Market. Reserves are not lent outside the banking system —there is no use of currency in the model. This implies that the aggregate holdings of reserves during the lending stage must equal the supply of reserves issues by the Fed:

$$\int_0^1 \tilde{c}_t(z) E_t(z) dz = M_t^0 \longrightarrow \tilde{c}_t \bar{E}_t = M_t^0.$$

Interbank Market. The equilibrium conditions for the interbank market depend on γ^+ and γ^- , the probability of matches in the reserve market. These probabilities, in turn, depend on M^- and M^+ , the mass of reserves in deficit and surplus. During the lending stage, banks are identical replicas of each other —scaled by equity. Thus, for every value of $E_t(z)$, there's an identical distribution of banks short and long of reserves. The shock that leads to $x = 0$ is $\omega^* = (\tilde{C}/p - \rho\tilde{D}) / (1 - \rho)$. This implies that the mass of reserves in deficit in is given by:

$$M^- = \mathbb{E}[x(\omega) | \omega > \omega^*] \left(1 - F\left(\frac{\tilde{C}/p - \rho\tilde{D}}{(1 - \rho)}\right) \right) \bar{E}_t$$

and the mass of surplus reserves is,

$$M^+ = \mathbb{E}[x(\omega) | \omega < \omega^*] F\left(\frac{\tilde{C}/p - \rho\tilde{D}}{(1 - \rho)}\right) \bar{E}_t.$$

Money Aggregate. Deposits constitute the monetary creation by banks, $M_t^1 \equiv \int_0^1 \tilde{d}_t(z) E_t(z) dz$. The endogenous money multiplier is $\mu_t = \frac{M_t^1}{M_t^0}$.

The definition of equilibrium is:

Definition. Given M_0, D_0, B_0 , a competitive equilibrium is a sequence of bank policy rules $\{\tilde{c}_t, \tilde{b}_t, \tilde{d}_t, \text{div}_t\}_{t \geq 0}$, bank values $\{v_t\}_{t \geq 0}$, government policies $\{\rho_t, D_{t+1}^{Fed}, B_{t+1}^{Fed}, M_{t+1}^0, T_t, \kappa_t, r_t^{ER}, r_t^{DW}\}_{t \geq 0}$, aggregate shocks $\{\Theta_t, F_t\}_{t \geq 0}$, measures of equity distributions $\{\Gamma_t\}_{t \geq 0}$, measures of reserve surpluses and deficits $\{M^+, M^-\}_{t \geq 0}$ and prices $\{q_t, p_t, r_t^{FF}\}_{t \geq 0}$, such that: (1) Given price sequences $\{q_t, p_t, r_t^{FedFunds}\}_{t \geq 0}$ and policies $\{\rho_t, D_t^{Fed}, B_{t+1}^{Fed}, M_{t+1}^0, \kappa_t, r_t^{ER}, r_t^{DW}\}_{t \geq 0}$, the policy functions $\{\tilde{c}_t, \tilde{b}_t, \tilde{d}_t, \text{div}_t\}_{t \geq 0}$ are solutions to Problem 4. Moreover, v_t is the value in Proposition 3. (2) The money market clears: $\tilde{c}_t \bar{E}_t = M_t$. (3) The loan market clears: $I_t^D = \Theta_t^{-1} q_t^{\frac{1}{\epsilon}}$, (4) Γ_t evolves consistently with $e_t(\omega)$, (5) the masses $\{M^+, M^-\}_{t \geq 0}$ are also consistent with policy functions and the sequence of distributions F_t . All the policy functions of Problem 4 satisfy $X = xE$.

Before proceeding to the analysis of particular parameterizations of the model, we discuss some

of its main features.

2.7 Real Side-Closing the Model

The competitive equilibrium defined above assumes an exogenous demand for loans, given by (9), an exogenous supply of deposits –banks face a perfectly elastic supply of deposits at rate R^D –and an exogenous tax T_t on the Fed budget constraint (??). We discuss in this section how we can extend our model to endogenize these objects. Details are provided in Appendix E.

Derivation of Loan Demand— In the Appendix ??, we derive 9 from a static problem of a firm that needs to borrow working capital loans. In particular, every period there is a continuum of firms that are created which..... but clearly there are many ways to obtain similar demand functions.

ALSO NEED TO ADDRESS TRANSFER WHICH SHOW UP IN HOUSEHOLD BUDGET CONSTRAINT market clearing conditions would be output=consumption by bankers and households deposits supply Fed+households=deposits banks labor markets ²¹

2.8 Discussion - Model Features

Fed Policy Instruments. In our model, the Fed’s policy decisions include: its holdings of commercial bank deposits and loans, $\{D_t^{Fed}, B_t^{Fed}\}$, reserve requirements ρ_t , and the interests rates $\{r_t^{ER}, r_t^{DW}\}$. Thus, the Fed here makes more decisions than the monetary authority in classical monetary models.²² Therefore, a natural question is whether the Fed in our model has additional tools than in those models. An analogy with models where a money demand emerges is useful. In our setup, the combination of, ρ , and the corridor rates $\{r^{ER}, r^{DW}\}$ determine the marginal benefit of holding reserves. Thus, $\{\rho, r^{ER}, r^{DW}\}$ induces the banks’ demand for reserves—the analogue money demand in shopping time models.²³ Like in those models, the rate of money growth through open market will affect inflation, the inverse of R^C . Here, R^C is influenced by the Fed’s money supply rule, through open market operations or direct lending against deposits. Thus, the choice of $\{\rho, r^{ER}, r^{DW}\}$ induces a money demand that in equilibrium determines the Fed Fund’s rate and the Fed’s open market operations and lending policies affect inflation. This means that the Fed can in fact affect two targets, the interbank market rate and inflation because it has additional power by distorting the money demand—something that in classical models follows from a technology specification. However, this additional power is not without limits.

²¹In the appendix we assume that this transfers are in turn lump-sum transfers to household. This assumption guarantees that these don’t affect the demand for loans.

²²**FOOTNOTE**[For example, in standard cash-in-advance models (Lucas, Stokey) the monetary authority chooses the growth rate of the money supplies via lump-sum transfers. In the cash-less limit of the new-Keynesian model, the Fed targets a single interest rate. Sargent and Wallace, Sargent and Ljungqvist [10 doctrines].]

²³See for example, Alvarez et al. (2002).

Here, the Fed has limits in targeting some value of R^C because there is a sufficiently low return such that reserves are not held in equilibrium. That level is tied to the decision of $\{\rho, r^{ER}, r^{DW}\}$. In addition, our Fed has additional power because can also affect the aggregate supply of loans directly. This is the case because we endow the Fed with fiscal independence in the sense that its transfers are not restricted. If it were to satisfy budget balance, the Fed would face an additional constraint. In summary, our Fed has the ability to affect interest rates and inflation independently within certain limits.

Financial Frictions and Externalities. There are several financial frictions in the model. The first friction is the illiquidity of loans in the balancing stage —or equivalently, the lack of a market for state-contingent transfers of reserves. These frictions are inconsequential when the Fed sets $r^{ER} = r^{DW} = 0$ —when the economy is at the zero-lower bound. In this case, reserves are borrowed costlessly from the interbank market and the Fed supplies them elastically. In the following section, we show that when this is the case, the spread between loans and deposits vanishes —as long as the capital requirement constraint does not bind.

A second financial friction results from the capital requirement. This friction induces a wedge between the deposit rate and the lending rate. Without these frictions, the economy would converge to an economy without spreads, zero bank equity, and an infinite leverage ratio. Notice that all of these distortions are induced by the financial frictions in the model which are determined by policy distortions. Presumably, in practice, these policy distortions respond to deeper frictions left out of the model.

There are two externalities affecting directly the banking sector in our model. One is a pecuniary externality that arises because banks fail to internalize how their financial decisions affect the price of loans, which in turn affect the value of equity —when loans are long-maturity— and their leverage constraint.²⁴

The second is a congestion externality, associated with the over-the-counter market: banks do not internalize that their reserve holdings have an effect on the matching probabilities. In particular, in a market characterized by average surplus of reserves, an individual bank does not internalize that by hoarding more cash, it reduces the probability of finding a match for other banks in surplus —the probability of finding a match for banks in deficit remains equal to one. Conversely, in a market characterized by average deficit of reserves, an individual bank does not internalize that by issuing more loans and accumulating less cash, it reduces the probability of banks in deficit of finding a surplus in the inter-bank market. This implies that the decentralized equilibrium displays underlending if banks on average have excess cash, or overlending if banks have on average cash deficit.

²⁴See for example [Lorenzoni \(2008\)](#) and [Bianchi and Mendoza \(2013\)](#) for an analysis of this externality.

3 Theoretical Analysis

3.1 Liquidity Premia and Liquidity Management

In this section, we derive an endogenous liquidity premium on reserves and analyze how monetary policy affects this premium.

Bank Portfolio Problem. Fix any given state X . To spare notation, we suppress the argument X from the returns and portfolio weights in Problem 7 and leave this reference as implicit. We rewrite Problem 7 by inserting the budget constraint into the objective:

$$\Omega = \max_{\substack{w_d \in [0, \kappa] \\ w_c \in [0, 1+w_d]}} \left(\mathbb{E}_{\omega'} \left[\left(\underbrace{R^B}_{\text{Return to Equity}} - \underbrace{(R^B - R^C) w_c}_{\text{Cash Opportunity Cost}} + \underbrace{(R^B - R^D) w_d}_{\text{Arbitrage}} - \underbrace{R^X(w_d, w_c, \omega')}_{\text{Liquidity Cost}} \right)^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}}.$$

This expression shows that if banks hold no reserves or issue no deposits, they obtain a return of R^B as their return on equity. Issuing additional deposits provide a direct arbitrage of $R^B - R^D$, but they also expose the banks to higher liquidity costs given by $R^X(w_d, w_c, \omega')$. In turn, banks can reduce these liquidity costs by holding more reserves, although they also bear an opportunity cost given by the spread between the return on loans and the return on reserves, $R^B - R^C$.

Liquidity Premium. First order conditions with respect to cash and deposits yield:

$$w_C :: \quad R^B - R^C = \frac{\mathbb{E}_{\omega'} \left[(R_{\omega'}^E)^{-\gamma} R_c^X(w_d, w_c, \omega') \right]}{\mathbb{E}_{\omega'} (R_{\omega'}^E)^{-\gamma}} \quad (12)$$

and,

$$w_D :: \quad R^D - R^C = \frac{\mathbb{E}_{\omega'} \left[(R_{\omega'}^E)^{-\gamma} (1 - R_c^X(w_d, w_c, \omega')) \right] + \mu}{\mathbb{E}_{\omega'} (R_{\omega'}^E)^{-\gamma}}, \quad (13)$$

where μ is the multiplier associated with the capital requirement constraint.²⁵ These expressions deliver a liquidity premium, i.e., a difference between the return on loans and reserves. We rearrange (12) and define the stochastic discount factor $m' \equiv \text{div}(X') \frac{R_{\omega'}^E \mathbb{E}[1 - \text{div}(X)]}{\mathbb{E}[\text{div}(X)]}$ to obtain:

$$\begin{aligned} \underbrace{R^B - R^C}_{\text{Cash Opportunity Cost}} &= - \frac{\mathbb{E}_{\omega'} [m' \cdot R_c^X(w_d, w_c, \omega')]}{\mathbb{E}_{\omega'} [m']} \\ &= \underbrace{\mathbb{E}_{\omega'} [R_c^X(w_d, w_c, \omega')]}_{\text{Direct Liquidity Effect}} - \underbrace{\frac{\text{COV}_{\omega'} [m', R_c^X(w_d, w_c, \omega')]}{\mathbb{E}_{\omega'} [m']}}_{\text{Liquidity-Risk Premium}}. \end{aligned}$$

The expression above says that the excess return on loans, $R^B - R^C$, the opportunity cost of

²⁵We are ignoring the non-negativity constraints on deposits and loans because they are not binding in equilibrium. In addition, we are assuming that reserves are strictly positive.

holding reserves, equals the additional benefit of holding reserves $\mathbb{E}_{\omega'} [R_c^x(w_d, w_c, \omega')]$ adjusted by a risk-premium associated with the withdrawal shocks. The direct benefit, $\mathbb{E}_{\omega'} [R_c^x(w_d, w_c, \omega')]$, is the expected marginal reduction in liquidity costs when holding more reserves. The liquidity risk premium emerges because the stochastic discount factor varies with the withdrawal shock.

We obtain a similar expression for the spread between loans and deposits:

$$\underbrace{R^B - R^D}_{\text{Arbitrage}} \geq \underbrace{\mathbb{E}_{\omega'} [R_d^x(w_d, w_c, \omega')]}_{\text{Direct Liquidity Effect}} - \underbrace{\frac{\text{COV}_{\omega'} [m', R_d^x(w_d, w_c, \omega')]}{\mathbb{E}_{\omega'} [m']}}_{\text{Liquidity-Risk Premium}}$$

where an equality holds if $w_d < \kappa$. This expression states that excess return on loans equals the expected marginal increase in liquidity costs from raising a unit of deposits, $\mathbb{E}_{\omega'} [R_d^x(w_d, w_c, \omega')]$, plus a liquidity risk premium. In addition, when the capital requirement constraint is binding, there is a larger excess return as capital requirements prevent this difference from being arbitrated away.²⁶

Define a bank's reserve rate as $L \equiv (w_c/w_d)$. The following lemma states that liquidity costs are linear functions of w_d given a liquidity ratio:

Lemma 1 (Linear Liquidity Risk) *$\mathbb{E}_{\omega'} [R^x(w_d, w_c, \omega')]$ is homogeneous of degree 1 in w_d .*

This lemma implies that excess returns of loans and reserves increase proportionally to the amount of deposits. We can obtain an expression for the marginal expected benefit of holding additional reserves:

Lemma 2 (Marginal Liquidity Cost) *The marginal value of liquidity is:*

$$-\mathbb{E}_{\omega'} [R_c^x(1, L, \omega')] = \chi_b \Pr \left[\omega' \geq \frac{L - \rho}{(1 - \rho)} \right] + \chi_l \Pr \left[\omega' \leq \frac{L - \rho}{(1 - \rho)} \right].$$

This lemma implies that the marginal value of additional liquidity, $\omega^d \mathbb{E}_{\omega'} [R_c^x(1, L, \omega')]$, equals the expected interests payments from the interbank market. **jb: Notice also that the marginal value of liquidity becomes independent of the withdrawal shock if $r^{ER} = r^{DW}$.** We will use this and the previous lemma to derive more properties.

3.2 Limit Case I: Risk-Neutral Banks ($\gamma = 0$).

For $\gamma = 0$, the bank's objective is to maximize expected returns. Thus, his solves:

$$\Omega = R^B + \max_{\{w_d, w_c\}} (R^B - R^D)w_d - (R^B - R^C)w_c - \mathbb{E}_{\omega'} [R^x(w_d, w_c)].$$

²⁶These expressions are similar to other standard asset-pricing equations with portfolio constraints except for the liquidity adjustment. Thus, this expression may be employed in other empirical investigations. For example, during the financial crises of 2008-2009, interest rate spreads increased. Credit risks and lower capital are a common explanation. The formulae above suggest that liquidity risks could also explain part of these spreads.

By Lemma 1, we can factor w_d from the problem above to obtain:

$$\Omega = R^B + \max_{w_d} \underbrace{w_d}_{\text{Leverage Choice}} \left((R^B - R^D) + \underbrace{\max_L \left\{ -(R^B - R^C) L - \mathbb{E}_{\omega'} [\tilde{R}^X(1, L)] \right\}}_{\text{Liquidity Management}} \right)$$

subject to $\omega^d \in [0, \kappa]$ and $L \in \left[0, \frac{1 + \omega^d}{\omega^d}\right]$.

This reformulation shows that for risk-neutral bankers, their portfolio problem can be separated into two independent problems: an optimal liquidity management problem and a leverage choice. Given this decomposition, the return per unit of leverage becomes linear. Issuing deposits yields a direct return of $(R^B - R^D)$ that follows from the arbitrage between the lending and deposit rates. However, the L fraction of those issued deposits are used to purchase reserves. The optimal liquidity ratio trades off the opportunity cost of obtaining liquidity against the reduction in the expected illiquidity cost. Let L^* be the optimal liquidity ratio —the maximizer of the expression in curled brackets. That L^* satisfies:

$$\underbrace{(R^B - R^C)}_{\text{Liquidity Premium}} = - \underbrace{\mathbb{E}_{\omega'} [R_c^X(1, L^*)]}_{\text{Direct Liquidity Effect}} \quad (14)$$

which is consistent with the first-order condition (12) —when $m = 1$. Once L^* is determined, the problem is linear in w^d if $L \leq \frac{1 + w^d}{w^d}$ is non-binding. In equilibrium, $L \leq \frac{1 + w^d}{w^d}$ is non-binding because otherwise an equilibrium features no loans. This, in turn, is ruled out by the functional form on the demand for loans. We can show the following relationship between the liquidity premium and the rates of the corridor system:

Proposition 5 *In equilibrium, $r_t^{ER} \leq R^B - R^C \leq r_t^{DW}$.*

The proposition shows that the Fed's choice of the corridor rates impose restrictions on the spread of loans relative to reserves. In particular, the spread between loans and the interest on reserves is bounded by the width of the bands of the corridor system, which imposes a constraint on the implementation of Fed policies ²⁷

Now, define the return to an additional unit of leverage —the bank's levered returns:

$$R^{L^*} \equiv \underbrace{(R^B - R^D)}_{\text{Arbitrage on Loans}} - \underbrace{\left((R^B - R^C) L^* + R^X(1, L^*) \right)}_{\text{Optimal Liquidity Ratio Cost}}.$$

With this expression, an equilibrium for the case of $\gamma = 0$ is characterized as follows:

²⁷ Under risk aversion, a risk-premium adjustment would emerge and the loan-reserve spread could exceed the width of the bands. However, the corridor system would still impose bounds on the interest spread because the liquidity-risk premium is also affected by the width of the bands.

Proposition 6 (Linear Characterization) *When $\gamma = 0$, in equilibrium, $\Omega = R^B + \max \{ \kappa R^{L^*}, 0 \}$, and:*

$$w^{*d} = \begin{cases} 0 & \text{if } R^{L^*} < 0 \\ [0, \kappa] & \text{if } R^{L^*} = 0 \\ \kappa & \text{if } R^{L^*} > 0 \end{cases} \text{ and } \text{div} = \begin{cases} 0 & \text{if } \beta v \Omega > 1 \\ [0, 1] & \text{if } \beta v \Omega = 1 \\ 1 & \text{if } \beta v \Omega = 1 \end{cases}$$

In a steady state, $\beta v \Omega = 1, \text{div} = \Omega - 1$. There are two classes of steady states:²⁸ In this case, banks retain earning until they reach E_{ss} . Next, we specialize the model to deterministic shocks.

Case 1 (non-biding leverage constraint steady state ($\mu = 0$)). *The steady-state value of equity, E_{ss} , is sufficiently large such that for*

$R_{ss}^B = 1/\beta$ is feasible and the following conditions hold:

$$R^{L^*} = (1/\beta - 1/\beta^D) - ((1/\beta - R^C) L^* + R^X(1, L^*)) = 0, R^E = \frac{1}{\beta}.$$

Case 2 (binding leverage constraint steady state ($\mu > 0$)). *E^{ss} is such that for $w^{*d} = \kappa$: $R_{ss}^B > 1/\beta$ and,*

$$R^{L^*} = (R^B - 1/\beta^D) - ((R^B - R^C) L^* + R^X(1, L^*)) > 0, (R^B + \kappa R^{L^*}) = \frac{1}{\beta}.$$

jb: add here If $r^{DW} = r^{ER} = K > 0$ the marginal value of liquidity is independent of ω . This implies that in the case of risk neutrality, changes in second or higher order moments of the distribution, do not affect portfolio choices. This is not true when $r^{DW} > r^{ER}$ because the penalty is non-linear.

jb: MY PREFERENCE WOULD BE TO SUMMARIZE THIS PART BELOW IN ONE SENTENCE FOLLOWING RISK NEUTRALITY. IN particular, the issue of the liquidity premium being beyond the Fed rates, also apply here (not just for risk neutral), so we could put that result here, which follows directly from the last proposition

3.3 Limit Case II: No Withdrawal Shocks ($\Pr(\omega = 0) = 1$).

A special case of interest is when there are no withdrawal shocks, $\Pr[\omega = 0] = 1$. Without withdrawal shocks there is no uncertainty. Hence, there is no difference between the portfolio decisions of risk-neutral and a risk-averse banker —although their dividend policies do differ. Thus, without uncertainty, the value of the portfolio problem is:

$$R^B + \max_{w_d \in [0, \kappa]} w_d \left[(R^B - R^D) + \left(\max_{L \in [0, \frac{1+\omega^d}{\omega^d}]} - ((R^B - R^C) L + \chi(L - \rho)) \right) \right].$$

²⁸Transition towards a steady state would be instantaneous as in other linear models —i.e., [Bigio \(2010\)](#) for example, unless there are binding equity constraints.

An equilibrium with deterministic shocks will satisfy the following analogue of Proposition 6:

Proposition 7 *In equilibrium, $r_t^{ER} \leq R_t^B - R_t^C \leq r_t^{DW}$. Moreover, in equilibrium $L^* = \rho$. The value of the bank's portfolio is $\Omega = R^B + \kappa \max \{ (R^B - R^D) - (R^B - R^C) \rho, 0 \}$ and the banker's policies are:*

$$w^{d*} = \begin{cases} 0 & \text{if } R^B < R^D + (R^B - R^C) \rho \\ [0, \kappa] & \text{if } R^B = R^D + (R^B - R^C) \rho \\ \kappa & \text{if } R^B > R^D + (R^B - R^C) \rho \end{cases} \quad \text{and } w^{c*} = \rho w^{d*}.$$

According to this proposition, in equilibrium, a banker sets the liquidity ratio exactly to the reserve requirement ρ . When shocks are deterministic, banks control the amount of liquidity holdings by the end of the period. In that case, they choose zero holdings of reserves if they can either borrow them cheaply from the discount window, $r_t^{DW} \leq R^B - R^C$, or they would not hold loans if the interest rate on excess reserves exceeds $R^B - R^C$. In equilibrium, reserves and loans are made so $r_t^{ER} \leq R_t^B - R_t^C \leq r_t^{DW}$ is an implementability condition for the Fed's policy. Moreover, since banks hold a liquidity ratio of $L^* = \rho$, this exercise shows that the reserve requirements act like a tax on financial intermediation because for every deposit issued, banks maintain ρ in deposits which earn a lower return. The rest of the equilibrium can be characterized by Propositions 3 and 4.

3.4 Limit Case III: Zero-Lower Bound ($r^{DW} = r^{ER} = 0$).

Consider the zero lower bound as states where $r_t^{DW} = r_t^{ER} = 0$, and therefore $\chi_t(\cdot) = 0$.²⁹ In this case, the Fed eliminates all the liquidity risk from the interbank market.

Thus, Ω becomes:

$$\Omega = R^B + \max_{w_d} w_d \left((R^B - R^D) + \left(\max_{L \in [0, \frac{1+\omega^d}{\omega^d}]} (R^B - R^C) L \right) \right)$$

Clearly, an equilibrium with strictly positive holdings of both loans and reserves requires $R^B = R^C$. Thus, the asset composition of the individual bank's balance sheet is indeterminate. If in addition, capital requirements do not bind then $R^B = R^C = R^D$ so $\Omega = R^B + \kappa \max \{ R^B - R^D, 0 \}$. In summary,

Proposition 8 *An equilibrium at the zero-lower bound, $r_t^{DW} = r_t^{ER} = 0$, satisfies, $R_t^B = R_t^C \geq R_t^D$. The inequality is strict iff capital requirements are binding.*

Notice that at the zero-lower bound, the Fed does have effects on lending rates unless capital requirements are binding. By carrying out open-market operations and targeting a corresponding

²⁹Here, one could argue that banks could request to hold currency—as opposed to electronic reserves—if $r^{ER} < 0$. With $r^{DW} < 0$, banks would make infinite profits by borrowing from the Fed.

inflation rate, the Fed can increase lending. When constraints aren't binding, $R^B = R^C = R^D$ the Fed has no additional power. This observation implies that in order to observe an aggregate decline in lending, the model needs either a contraction in loan demand Θ_t if constraints aren't binding. If constraints are binding, a reduction in κ_t or a drop in equity may lead to a decline in lending. At the zero-lower bound, higher uncertainty in the interbank market plays no role.

4 Calibration

4.1 Dispersion of Deposit Growth

Calibrating our model requires a random-withdrawal process for deposits, F_t . To obtain an empirical counterpart for this shock, we use information from individual commercial-bank Call Reports collected by the Federal Deposit Insurance Corporation (FDIC). The Call Reports contain balance-sheet information from regulatory filings of US commercial banks. We define a period in our model to be one quarter. We use information from 2000Q1-2010Q4.

In the model, all banks experience the same expected growth rates in deposits every period. Deviations from the mean only respond to the withdrawal shocks, ω . Hence, deviations from average deposit growth rates can be mapped directly into F_t . Thus, to calibrate F_t we compute the distribution of cross-sectional deposit growth rates in the data and then approximate F_t by computing this distribution in terms of deviations from its mean. The units of observation for this approximation are quarter-bank observations for total deposit growth-rates in deviations from quarter means in the sample.

There is no obvious appropriate empirical candidate for the empirical counterpart for deposits. In our model, banks have only one form of liability: demand deposits. In practice, commercial banks have other liabilities that include forms of wholesale funding —bonds and interbank loans—, long-term deposits —time and savings deposits—, as well as demand deposits. We choose total deposits, which include time and saving deposits and demand deposits, as our empirical counterpart. There are several reasons for this choice. In contrast to demand deposits, total deposits feature a similar trend than the growth of all bank liabilities. Also, total deposits in the data are substantially less disperse than the counterpart for demand deposits and we do not want to attribute all deposit funding to demand deposits. In practice, total deposits may include savings for short periods of time, or may also be removed from a bank at a cost. Finally, there is a substantial dispersion in growth rates of deposits as Figure 1 shows. The histogram plots the empirical frequencies of the cross-sectional deviations of the growth rates for each quarter-bank observation from the average growth rate of the cross-section at a given quarter. The bars in Figure 1 report the pre-crisis frequencies for the 2000Q1-2007Q4 sample of cross-sectional dispersion in deposit growth rates. The solid curve is the analogue for the Great Recession—post-crisis—, 2008Q1-2010Q4 sample. The substantial dispersion in growth rates in Figure 1 also shows that

these measure is relevant to capture liquidity risk. The comparison among both samples shows only a minor change in the distribution during the crises with a slightly more concentrated mass at the lower tail. We use this information and a shut-down in the interbank market to study the holdings of reserves that potentially follow from a precautionary motive.

Given the constructed empirical distribution, we fit a logistic distribution $F(\omega, \mu^\omega, \sigma^\omega)$ with $\mu = XXX$ and $\sigma^\omega = XXX$. We conduct a Kolmogorov-Smirnov goodness-of-fit hypothesis test and do not reject that the empirical distribution is logistic with a 50 percent confidence. The Data Appendix, provides additional details on how we construct the empirical distribution of deposit growth-rate deviations. Our model also predicts that the behavior of equity should be perfectly correlated with the behavior of deposits. We report this correlation in Appendix ???. We find that the correlation is positive, as suggested by our model significantly lower from one which is reasonable given that equity captures variations in the prices of securities, credit risks and operating costs that we do not include in the model. The data appendix discusses this point as well as the validity of the growth independence assumption. A final thing to note is that the variation in deposit growth has shifted to the left.

4.2 Parameter Values

The values of all parameters are listed in Table ??. We assign values to eleven parameters $\{\kappa, \beta, \delta, \gamma, \epsilon, \rho, r^{DW}, r^{ER}, R^d\}$. We set the capital requirement, κ , and the reserve requirement, ρ , to be consistent with actual regulatory parameters. In particular, we set $\kappa = 10$, which corresponds to a required Capital Ratio of 9 percent, and $\rho = 0.05$. For now, we set $\delta = 0$ so that loans become one-period loans. We set risk aversion to $\gamma = 0.5$. We set $r^{ER} = 0$, which is the pre-crisis interest rate on reserved paid by the Federal Reserve. The interest rate on discount window is set to be 2.5 percent annually, which delivers a Fed funds rate of 1.25 percent.³⁰ The interest rate on deposits is set to $R^D = 1$. The value of the loan demand elasticity given by the inverse of ϵ is set to $= 2$, which is an estimate of the loan demand elasticity by Bassett et al. (2010). Finally, we set the discount factor so as to match a return on equity of 8 percent a year. This implies $\beta = 0.98$.

5 Steady State Equilibrium Portfolio

We start by analyzing the equilibrium portfolio at the stochastic steady state and investigate the effects of withdrawal shocks over banks' balance sheets. The equilibrium portfolio corresponds to the solution of the Bellman equation (1) evaluated at the loan price that clears the loans market, according to condition (11), and the equilibrium probability of matching in the inter-bank market.

³⁰Since we consider a steady state without inflation, this is also the real interest rate.

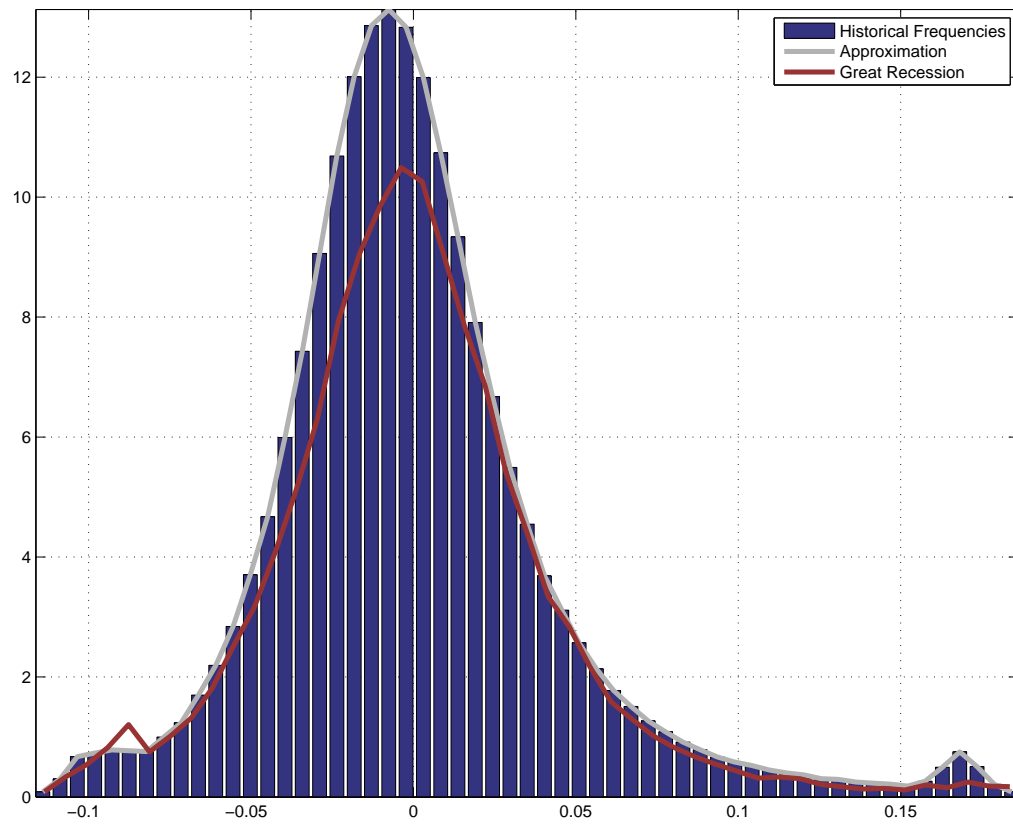


Figure 1: Cross-Sectional Distribution of Deviation from Cross-Sectional Average Growth Rates

The left panel of Figure 2 shows the probability distribution of the deficit in cash in the balancing stage, and the penalty associated with each level of deficit –the mass of the probability distribution is rescaled to fit in the same plot. The penalty χ has a kink at zero, due to the fact that the discount window rate is larger than the interest rate on excess reserves. This asymmetry in the return for cash is a crucial feature of our model. Notice that the distribution of the cash deficit inherits the distribution of the withdrawal shock, as the cash deficit depends linearly on the withdrawal realization. Because in equilibrium, there is on average excess surplus, the distribution’s mean is located to the right of zero. In particular, it is **jb1 XXX** percent more likely that a bank will end up with positive surplus.

The right panel of Figure 2 shows the distribution of equity growth as a function of the withdrawal shock. In equilibrium, the banks that experience deposit inflows will increase the size of their equity, whereas those that experience deposit outflows will tend to reduce the size of their equity. Because of the non-linear penalty that inflicts relatively higher losses when adverse withdrawal shock hits the bank, the distribution of equity growth is skewed to the left. In particular, there is a fat tail with probabilities of losing about 2 percent of equity in a given period, while the probability of growing more than 1 percent in a period is close to nil.

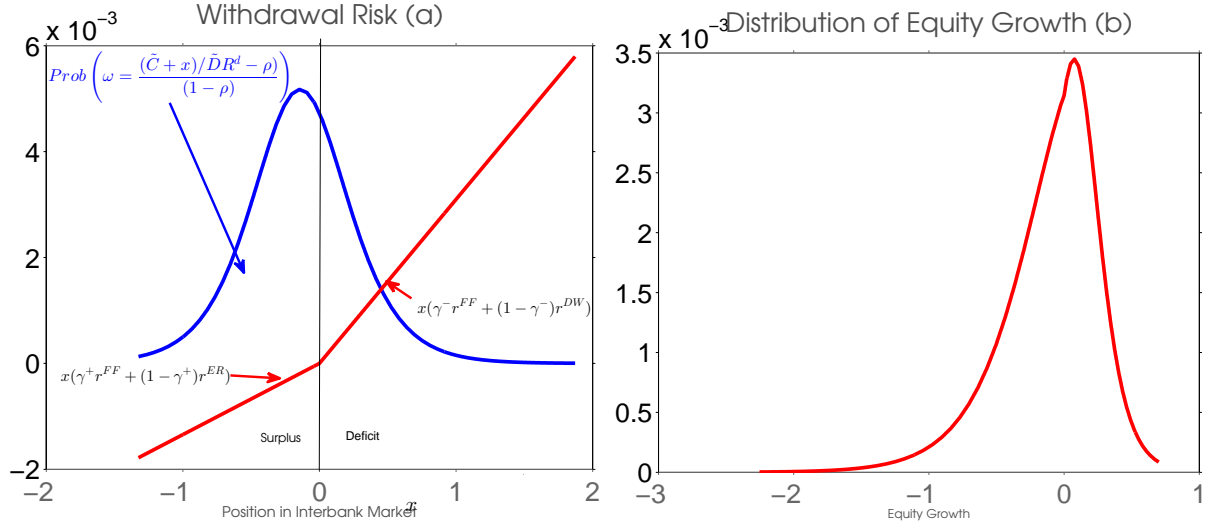


Figure 2: Portfolio Choices and Effects of Withdrawal Shocks

6 Policy Functions - Prices Given

We start with a partial equilibrium analysis of the model by showing banks policy functions at different loan prices. Figure 3 reports decisions for cash, loans, dividend, as well as liquidity and leverage ratios, the value of the asset portfolio, liquidity risk, expected returns and expected equity growth for different levels of loan prices. These policies correspond to the solution to the Bellman equation (4) for different values of loan prices q , and fixing the probability of a match

in the interbank markets at its steady state value. The solid dots in Figure 3 corresponds to the values associated with the steady state price of loans.

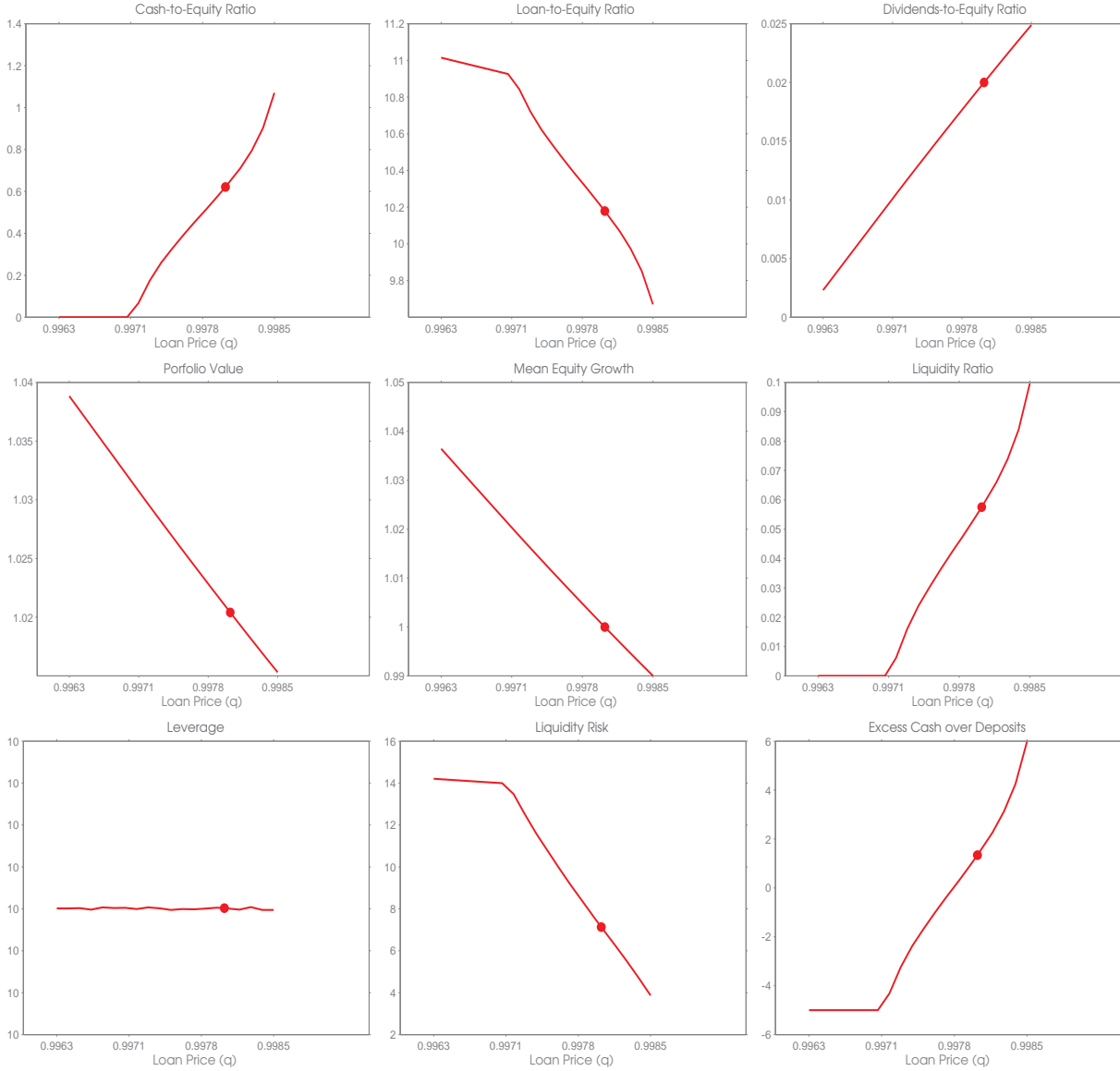


Figure 3: Policy function for different Loan Prices

As Figure 3 shows, the supply of loans is decreasing in loan prices, i.e., increasing in the return on loans, whereas reserve rates are *increasing* in loan prices. As loan prices decrease, loans become relatively more profitable leading banks to keep a lower fraction of its assets in relatively low return assets, i.e., cash. Notice also that for a sufficiently low price of loans, the non-negativity constraint on cash becomes binding.

In addition, dividends are increasing in loan prices due to a substitution effect: when return on loans are high, banks cut on dividend rate payments to allocate more funds to profitable lending. Exposure to liquidity risk, measured as the standard deviation of the cost from rebalancing the portfolio χx , is also decreasing in loan prices, reflecting the fact that banks' asset portfolio becomes

relatively more illiquid when loan prices decrease.

7 Transitional Dynamics

This section studies the transitional dynamics of the economy in response to different shocks associated with hypotheses 1–5. The shocks we consider are equity losses, a tightening of capital requirements, disruptions in interbank markets, increases in the dispersion of withdrawals, credit demand shocks, and changes in the discount window and interest on reserves. Shocks are unanticipated upon arrival at $t = 0$ but their paths are deterministic for $t > 0$.³¹ In each exercise, the Fed has a zero inflation target.³²

7.1 Equity Losses

We begin with a shock that translates into a sudden unexpected decline in bank equity. This shock captures an unexpected rise in non-performing loans, security losses or to off-balance sheet items that are left out of the model.³³ Figure 4 illustrates how banks's balance sheets shrink in response to equity losses of 2 percent. The top panel shows the evolution of total lending, total cash and liquidity risk, and the bottom panel shows the level of equity, return on loans and the dividend rate.

To understand the dynamics of the model, recall that all bank policy functions are linear in equity. Thus, holding prices fixed, a loss in equity leads to a proportional 2 percent decline in the quantity of loans supplied and reserve holdings. However, the contraction in loans supply does generate a drop in loan prices on impact—a movement along the demand for loans. This reduction further reduces the supply of credit: the capital requirement constraint applies to marked-to-market equity, which falls with prices.

The reduction in loan prices also leads to an increase in loan returns throughout the transition. As a consequence of the higher profitability on loans, reserve holdings fall relatively more than loans. Banks shift their portfolios towards loans and desirably expose themselves to more liquidity risk. The overall return to the banks portfolio also increases. With this, dividends fall as banks see their opportunity cost increase. The increase in bank returns and lower dividends leads to a gradual recovery of original equity losses. As equity recovers, the economy converges to the initial

³¹The assumption of unanticipated shocks is mainly for pedagogical purposes. In fact, it is relatively straightforward to extend the model to allow for aggregate shocks, which are not unanticipated. Due to scale invariance, we would not have to keep track of the cross-sectional distribution of equity.

³²We assume this not only for illustrative reasons but also because in the context of the Great Recession, the core personal consumption expenditures index (PCE) remained close to 1 percent.

³³One way to incorporate this explicitly in the model would be to consider a bank specific shock to the default rate on loans. To the extent that equity is the only state variable, the analysis of the transitional dynamics is analogue to studying the evolution of the model with a richer structure on loans.

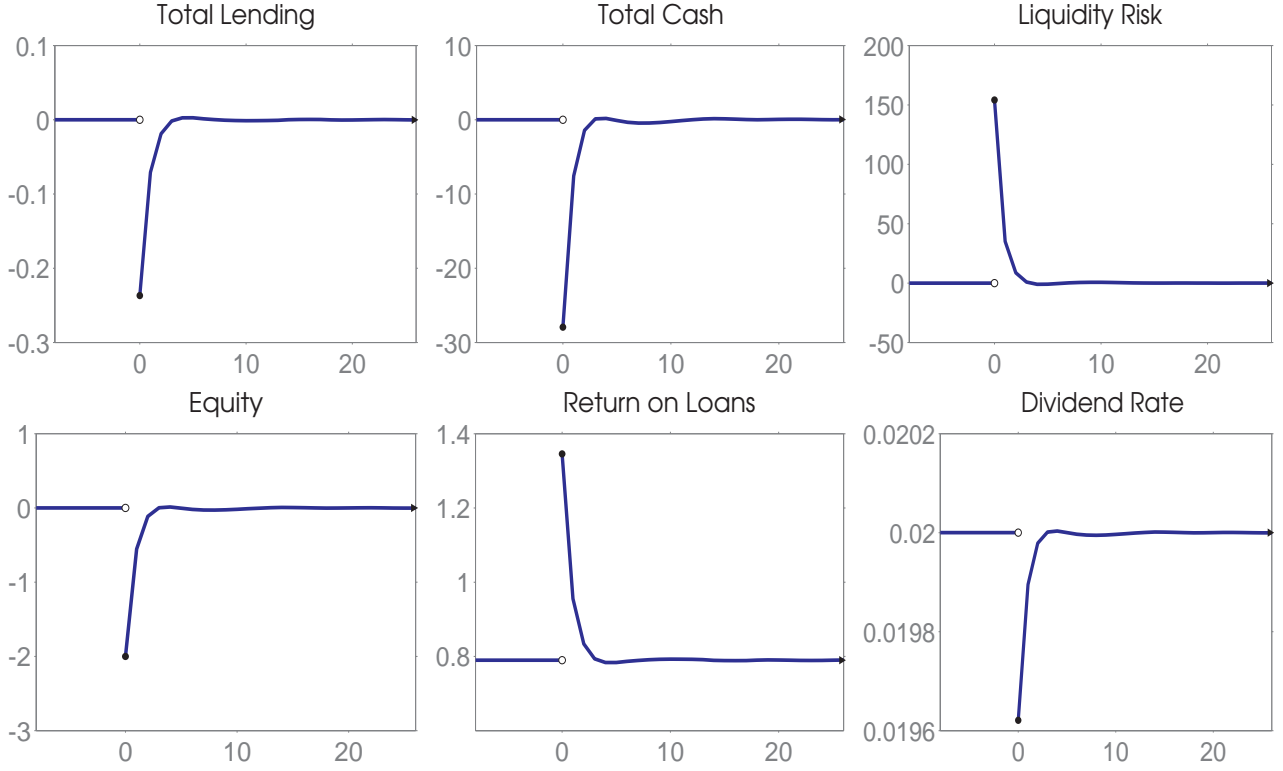


Figure 4: Impulse Response to Equity Losses

steady state where the returns to equity are paid as dividends. The transition is quick. The effects of the shock cannot be observed after six quarters.

7.2 Capital Requirements

The effects of sudden and permanent tightening of capital requirements, i.e., a reduction in κ are shown in Figure 5. The shock is a 10 percent decrease in κ which is associated with a 1 percent increase in the capital ratio of banks for the calibrated level of leverage. The short-run behavior of the transition is very similar to the behavior after equity losses. As with equity losses, the contraction in capital requirements reduces the supply of loans as banks funding constraint gets tighter—the capital requirement is binding in steady state. There is also a “second round” tightening in the capital requirement constraint as q_t decreases.

On impact, reserve holdings fall for two reasons. First, the increase in the return on loans increases that follows from the contraction in loans supply increases the opportunity cost of holding reserves. Second, because banks have lower leverage, this reduces the liquidity risk premium. Having a lower liquidity risk premium implies that banks are willing to expose more to liquidity risk. The increase in the return on loans leads banks to reduce dividends, which contribute to the build up of their equity over time. In the long-run, equity converges to a higher value such that the return to the banks portfolio is the same as in the initial steady state. The

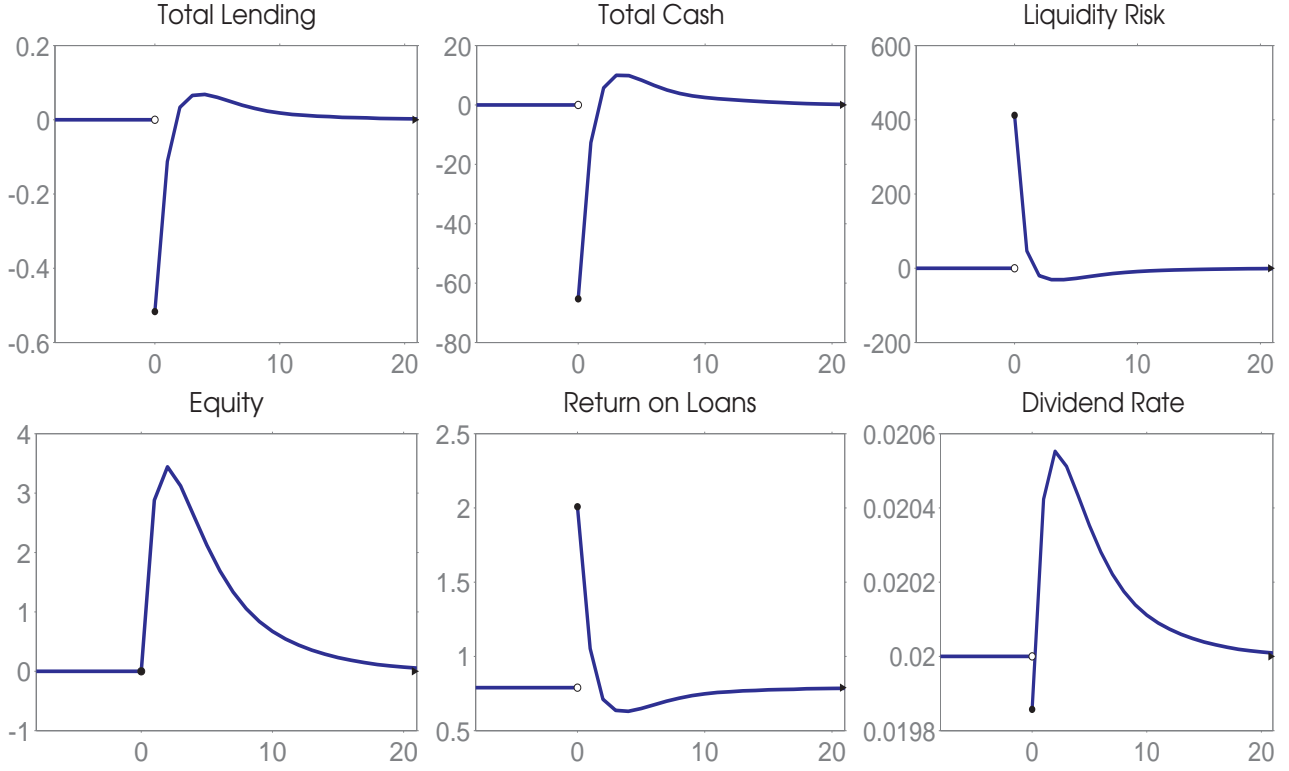


Figure 5: Impulse Response to a Tightening in Capital Requirements

holdings of reserves relative to loans change at the new steady state because of the reduction in the liquidity risk premium. The return on loans converges to a lower value, similarly, as the risk-premium falls with less leverage.

7.3 Increase Precautionary Holdings

7.3.1 Bank-runs

Here we study the possibility of a bank-run. In other words, we consider a 5 percent probability that all the deposits are withdrawn from a bank—that is $\omega = 1$. We maintain the assumption that deposits are not withdrawn from the banking system as a whole.³⁴ We assume that this bank-run probability follows a deterministic AR(1) process such that the shock lives for about 2 years. The effects of this shock are illustrated in Figure 6.

The risk of a bank-run generates an increase in liquidity risk, leading banks to hoard cash. Notice that liquidity risk is still about 3 times as large as in the initial steady state. Notice that this occurs eventhough banks hold more reserves and the Fed supplies reserves to keep the price constant. The reason for the increase in liquidity costs follows from the asymmetry in the liquidity cost function and the response of the return on loans. Indeed, the increase in liquidity risk spills over to the loans market. Higher liquidity costs induce a decline in the supply of loans.

³⁴Thus we adjust F accordingly by assuming a 5 percent probability of a large inflow of deposits.

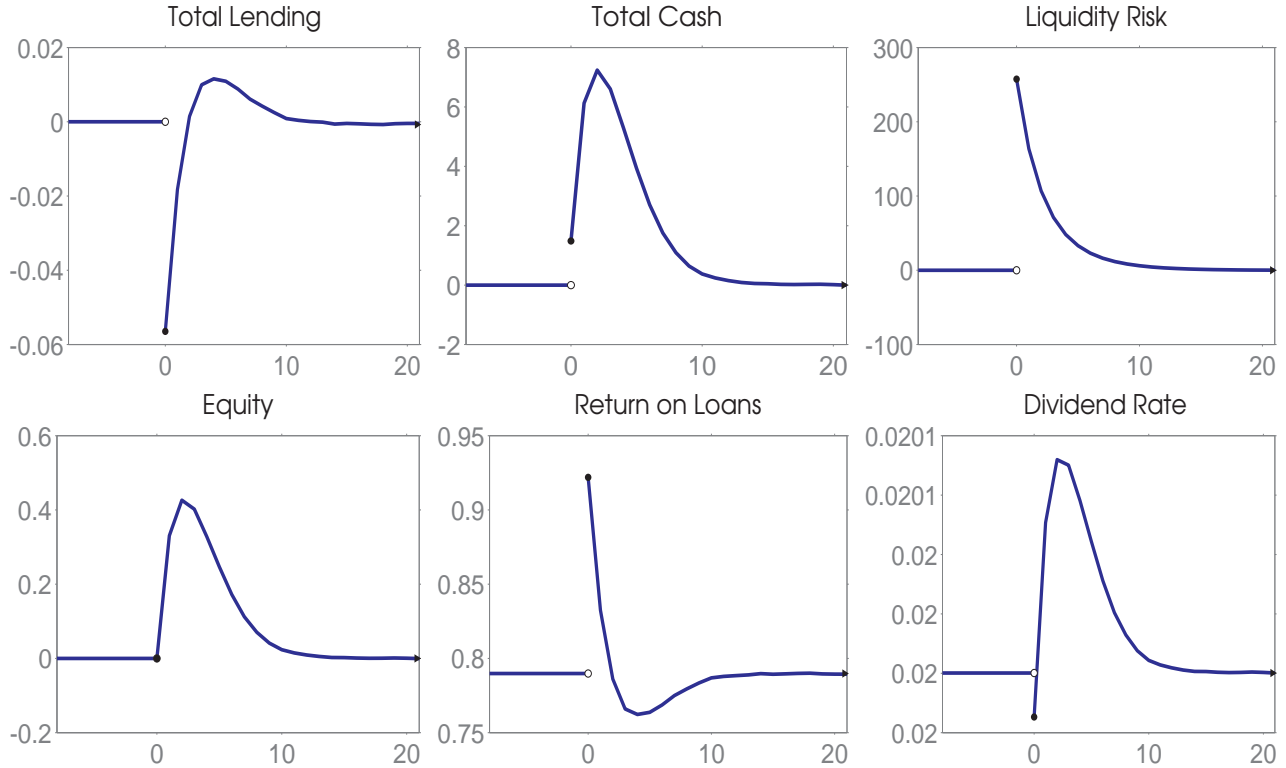


Figure 6: Impulse Response to a Bank-Run Shock

In equilibrium, this leads to an increase in the price of loans and a decline in the aggregate volume of lending—a movement along the loans demand schedule. The increase in the return weakens the desire to hold reserves to reduce the exposure to liquidity risks.

In tandem, banks respond to the risk of a bank-run by cutting dividend payments. Although higher liquidity costs are associated with lower returns, the contraction in loans supply generates an increase in expected bank returns that leads to an increase in bank returns. The reduction in dividend payments and returns on equity lead to an increase in equity over time. As equity grows, this mitigates the fall in lending ratios. Eventually lending raises above the steady state value because the effect on bank equity eventually compensates for the portfolio effect.

7.3.2 Interbank Market Shutdown

Disruptions in the interbank market can be studied through a shock that forces the probability of a match in the interbank markets to be zero—under this shock, the interbank market is not active.³⁵ Hence, reserves are borrowed or lent only to the Fed. In particular, banks that face a reserve deficit borrow directly from the Fed at r^{DW} . Thus, the shock increases liquidity costs. The effects of the interbank market freeze is shown in Figure 7. Overall, the effects are similar to the bank-run shock we study above.

³⁵A recent macroeconomic model of endogenous interbank market freezing due to asymmetric information with one-period lived banks is Boissay et al. (2013).

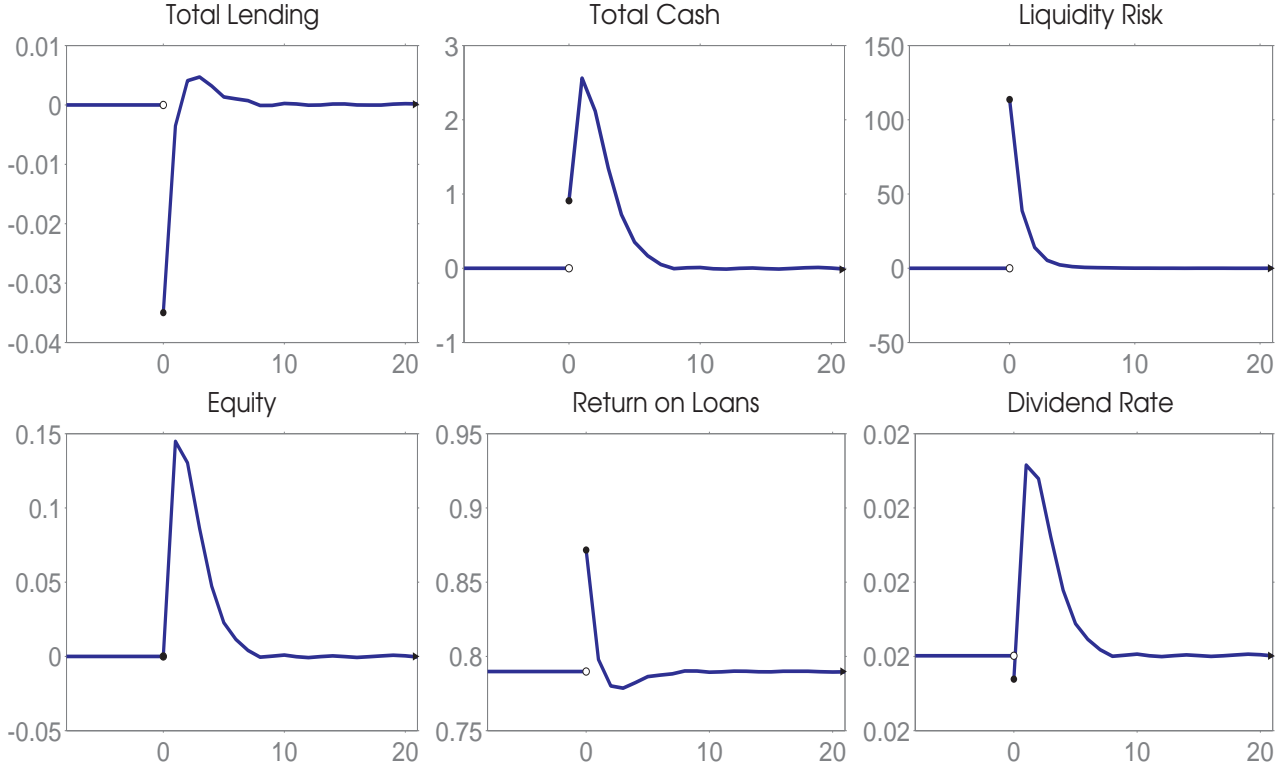


Figure 7: Impulse Response to Freezing in Inter-bank Markets

7.4 Credit Demand

The effects of negative credit demand shock are captured through a decline in Θ_t , the log-intercept of the loans demand. In the microfoundation we provide in the appendix, this shock captures a decline in total factor productivity or an increase in labor market distortions that reduce the demand for working capital by firms.³⁶ Figure 8 illustrates the effects of a negative temporary shock to credit demand. We assume the shock follows deterministic AR(1) process that lasts for about 7 years.

The effects of credit demand shocks contrast sharply with the effect of the shocks considered above because all of the shocks above cause a contraction in the supply of loans. In this case, the demand shock causes a decline in the return to loans –and a shift along the supply curve. As a result, banks shift their portfolios towards reserves as the opportunity cost of holding reserves is lower. The liquidity risk drops almost vanishes. Initially, banks also respond by paying higher dividends due to the overall decline in the return of their portfolios. The reduction in returns and increments in dividend payments brings equity significantly below steady state. As the credit demand shock dies out —around a year and a half later— the economy follows a similar transition as with the shock to equity slowly increasing lending rates and reducing dividend rates until equity

³⁶More broadly, a credit demand shock could also have a financial origin, e.g., a decline in the value of firm's or household collateral that limit their ability to borrow. Moreover, this shock is also isomorphic to a reduction in credit-quality if banks are well diversified.

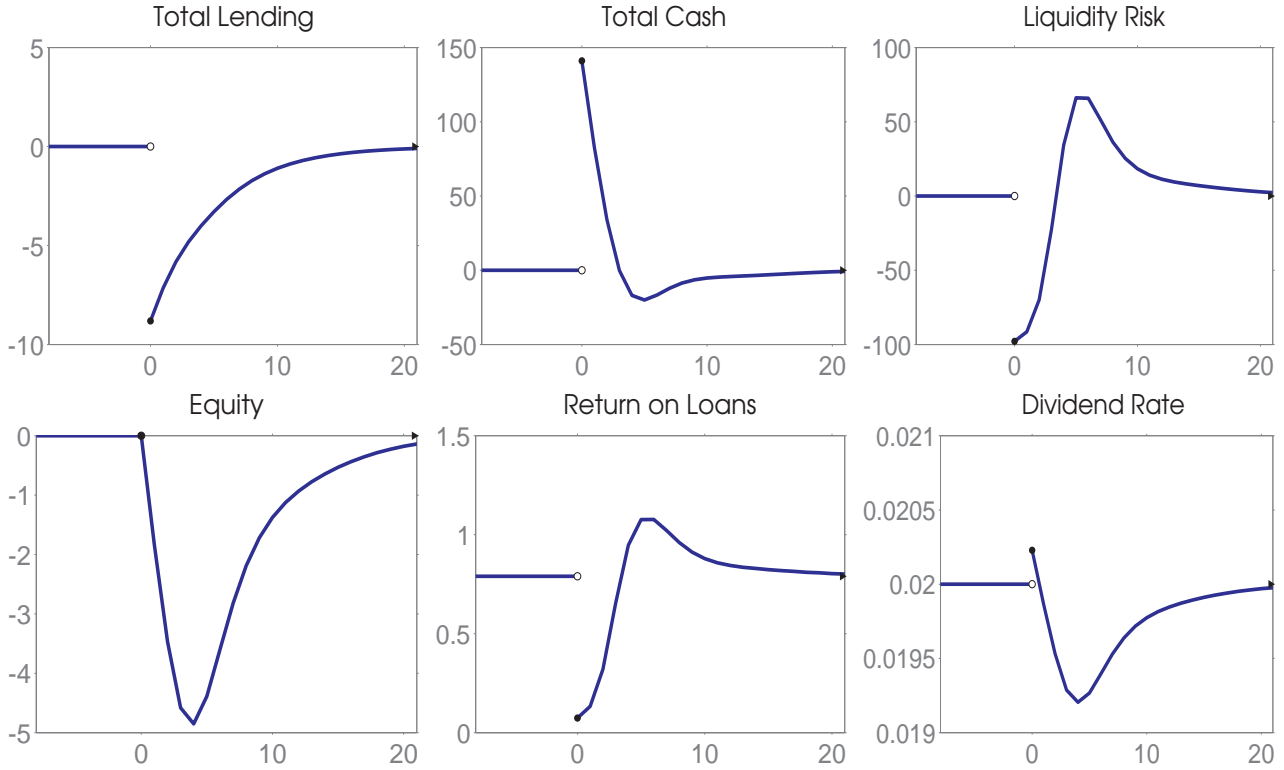


Figure 8: Impulse Response to Credit Demand Shock

reaches its steady state level.

7.5 Discount Window and Interest on Reserves

We now analyze the effects of interest rate policy shocks. In the experiment, we study a shock to the discount window rate of 100bps—in annualized terms. The effects of a rise in the discount window rate are shown in Figure 9. Banks respond to the increase in the discount window rate by reducing lending. The policy effects are very similar to the effects of shocks that increase the liquidity costs.

A shock to the interest on excess reserves works similarly. We study a shock that raises from 0 to 100bps, a shock that corresponds to the recent Fed of remunerating excess reserves. The effect of this policy are illustrated in Figure 10. As the central bank pays interest on excess reserves, cash becomes relatively more attractive. In turn, banks reallocate their portfolio from loans to cash.³⁷

³⁷Notice that liquidity risk does not decline despite the increase in cash holdings by banks. This occurs because the increase in the interest rate on excess reserves, leads to larger differences in returns between banks on surplus and deficit.

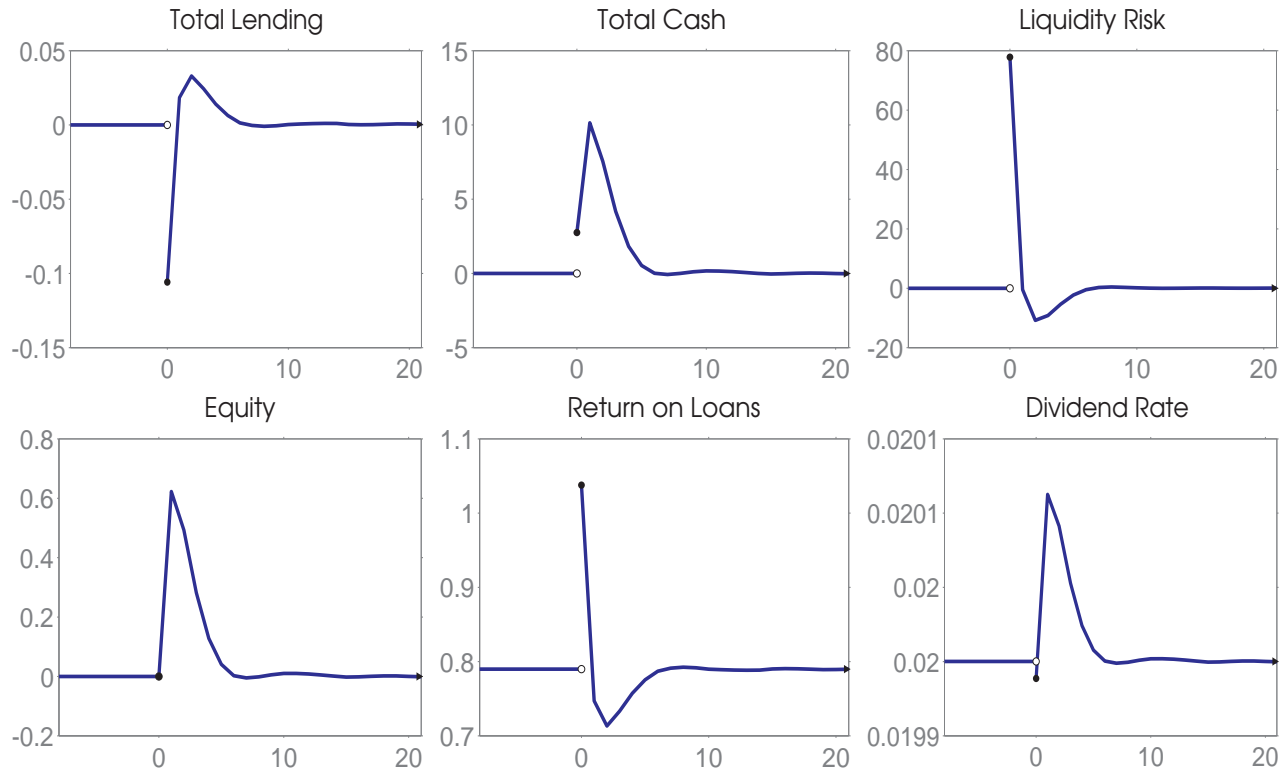


Figure 9: Impulse Response to Rise in Discount Window

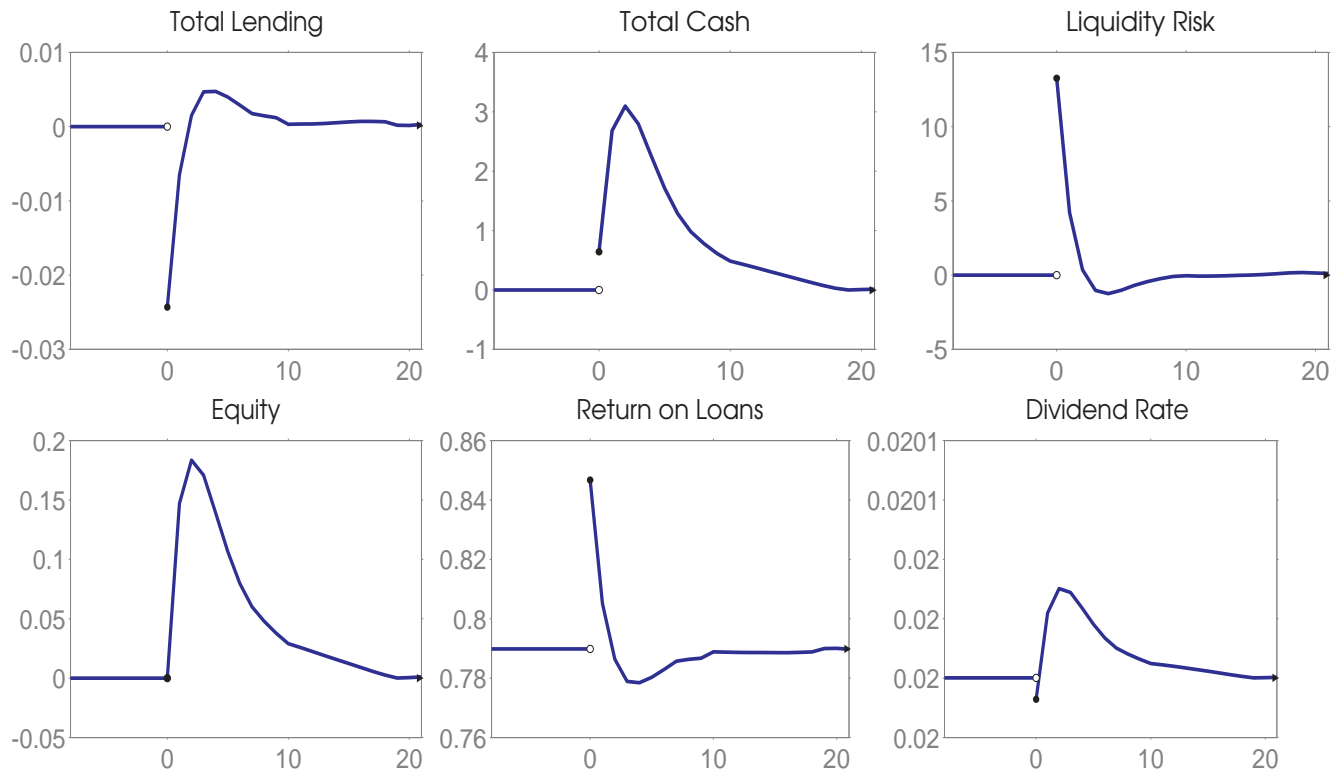


Figure 10: Impulse Response to Interest on Reserves

7.6 Unconventional Open Market Operations

Finally, we study purchases of loans by the Fed. We study the effects of loan purchases amounting to 2 percent of the steady state outstanding stock of loans. We assume that the Fed gradually reverses the operation in approximately 4 years. Unconventional open-market operations boost total lending in the economy, as shown in Figure 11. However, there is a partial crowding out effect. The purchases by the Fed lowers the return on loans, which in turn leads private banks to lend less. In equilibrium, banks also end up holding more reserves. As a result, the transitions are similar the transitions after negative credit demand shock with the difference that total bank lending increases because of the Fed's holdings.

8 Application - Which Hypotheses Fit the Crisis Facts?

This section explores the possible driving forces that explain the holdings of excess reserves without a corresponding increasing in lending by banks sector during the US financial crisis. Here, we discuss how the different shocks we studied in the preceding section fit the patterns we observe for the data. As a parenthesis, we first revisit some key fact about monetary policy, monetary aggregates and banking indicators during the recession.

8.1 Monetary Facts

Fact 1: Anomalous Interbank Interest Rate Behavior. The top-left panel of Figure 12 shows how the daily series of the Fed funds rate fluctuates around the Fed's target —the flat series. The Fed's corridor system is determined by the overnight discount rate and the interest rate on excess reserves, the analogue of $\{r_t^{DW}, r_t^{ER}\}$ in our model. Prior to the Great Recession, the Fed funds rate was consistently within these bands —prior to 2008 the Fed later rate was zero. During the midst of the crisis the Fed funds rate exceeded the discount rate. This anomalous behavior reflects the disruptions in the interbank market that we try to capture with the shocks to the interbank market. Since the beginning of the recession, the Fed funds rate has been brought down to its lowest historical levels for almost 5 years.

Fact 2: Fed Balance-Sheet expansion. The top-middle panel of Figure 12 shows the assets held in the Balance Sheet of the Fed. The picture shows a substantial increase in asset holdings that corresponds to the large scale open-market operations programs carried out after the collapse of Lehman Brothers. During the initial face, the increment in assets where direct forms of lending to banks. The subsequent programs included unconventional open market operations such as the purchase of long-term bond and mortgage-backed securities. The top-right panel shows the increase in the Fed's assets relative to the total of bank credit for commercial banks in the US. Whereas prior to the crisis, this stabilized close to 10%, this ratio has reached 40% by the end

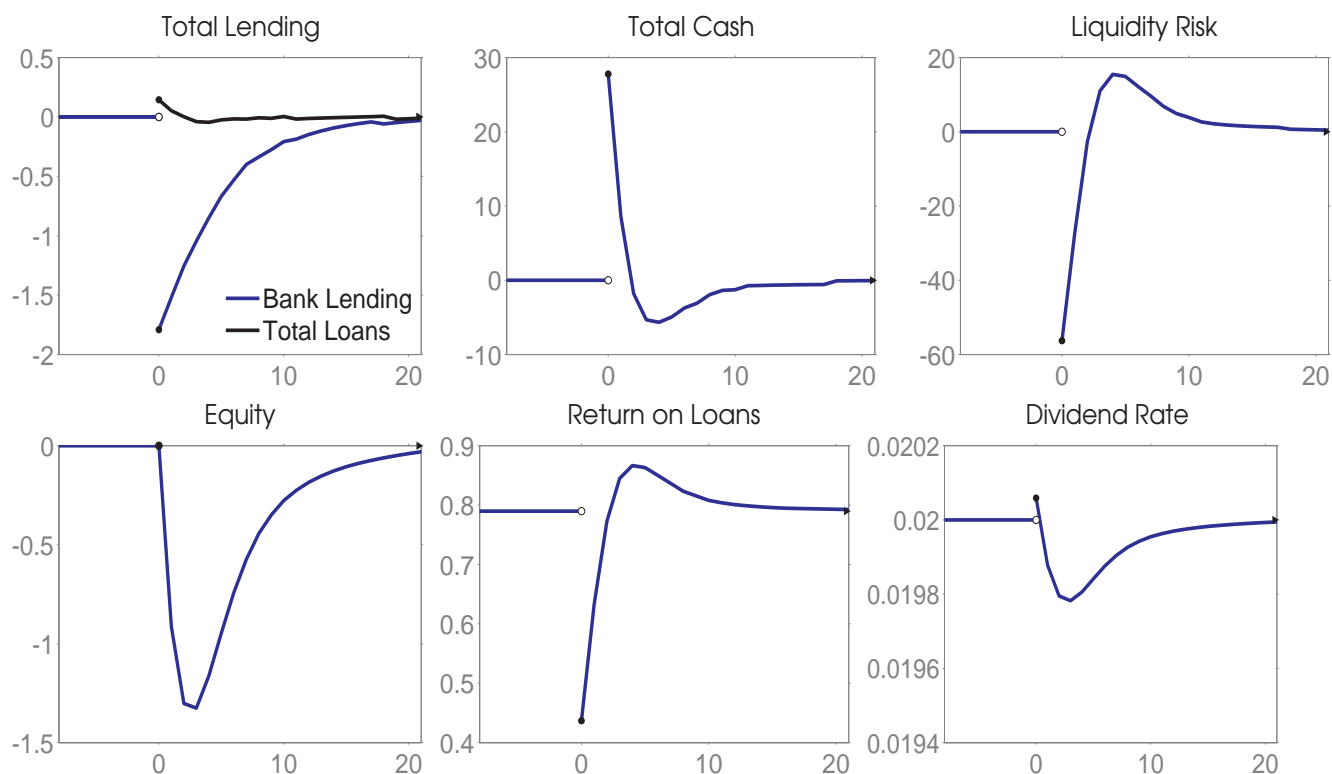


Figure 11: Impulse Response to Unconventional Monetary Policy

of 2013. The counterpart of this figure is a proportional increase in Fed's liabilities, Fed reserves. The model captures this increase through the Fed's purchases of loans.

Fact 3: Excess-Reserve Holdings. The bottom left panel of Figure 12 shows an increment in holdings of excess reserves by the banking system. Whereas pre-crisis, there were virtually no excess reserves, post crisis, excess reserves amount for 16 times the amount of reserves —during the period, there is a slight increment in required reserves. The increment in excess reserves —and not required reserves— shows that there has not been an equivalent expansion in lending —deposit creation— by commercial banks.

Fact 4: Depressed Lending Activities. The bottom-middle panel of Figure 12 shows a decline in commercial and industrial lending during the crisis. The figure shows the raw series and a series that subtracts the decline in the stock of loan commitments from the original series. This figure shows that while there was a large monetary expansion during the middle of the crisis, there was a simultaneous substantial decline in lending activities at the time.

Fact 5: Drop in Money Multiplier. The large drop in the money multiplier for M1 is a summary of facts 2, 3, and 4.

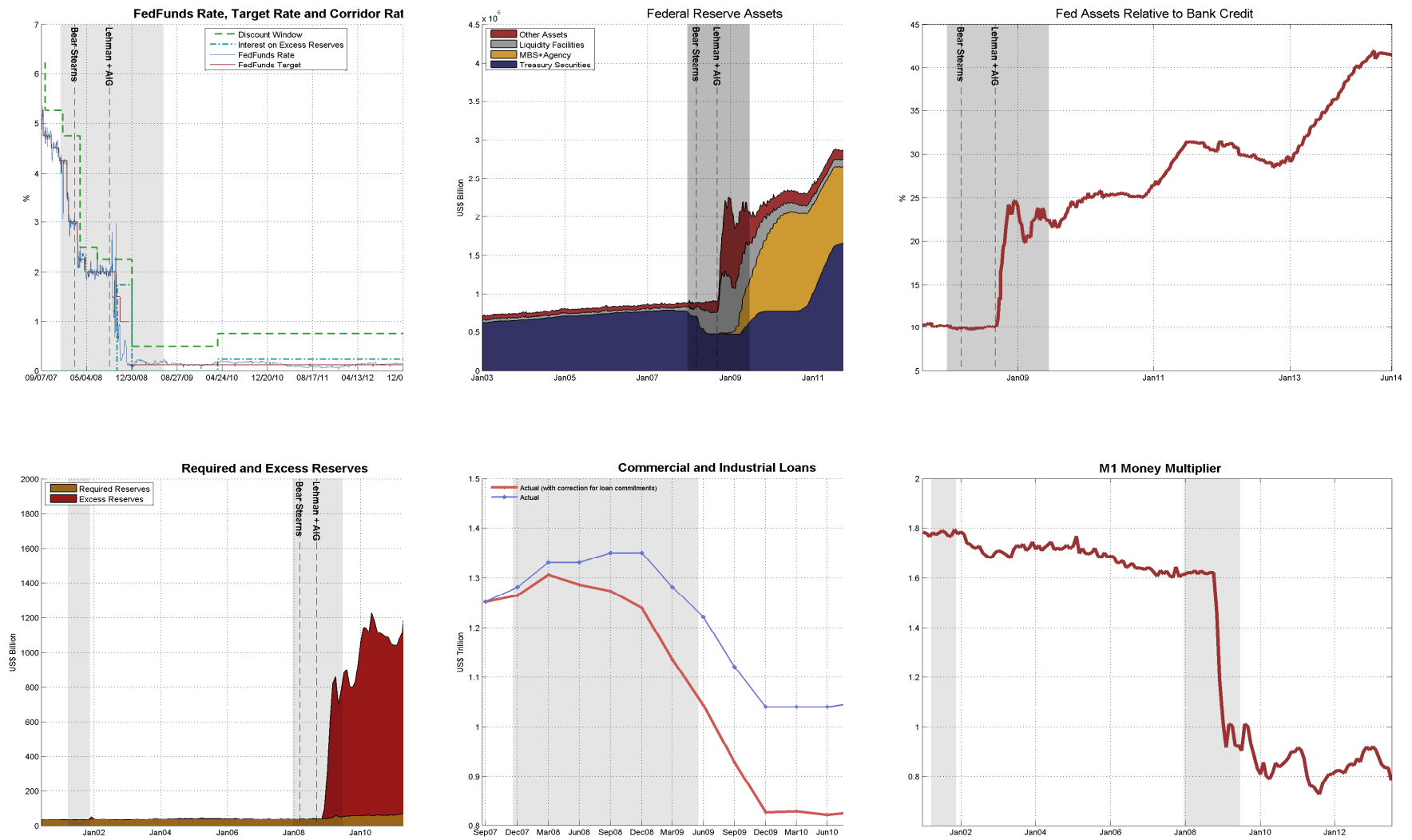


Figure 12: Monetary Facts

8.2 Banking Facts

Through the model, the behavior of banking indicators ratios reveal the nature of the shocks that affected banks during the crisis. All the series in this section correspond to Commercial Banks. The series are computed from Bank Call Reports and ratios are reported in averages and averages weighted by assets.

Fact 6: Decline in Book Leverage. The upper-left panel of Figure 13 shows the decline in the tangible leverage —measured using only tangible assets— of banks at book-value. The average leverage falls from 16 to about 12 by the end of the crisis.

Fact 7: Increase Liquidity Ratio. The upper-right panel of Figure 13 shows the behavior of the liquidity ratio —liquid assets over illiquid assets. The data shows an increase from 6% to 12%.

Fact 8: Bank-Equity Losses. The bottom-left panel of Figure 13 shows the behavior of the realized returns to equity of banks. The figure shows a sharp decline, especially concentrated among the largest banks. Losses exceeded 5% during the peak of the crisis.

Fact 9: Dividends. The bottom-right panel of Figure 13 plots the evolution of the dividend-per-equity of banks. Dividend declined all throughout the duration of the recession, but erratically increased thereafter.

The discusses which combination of shocks are consistent with these patterns.

Banking Indicators

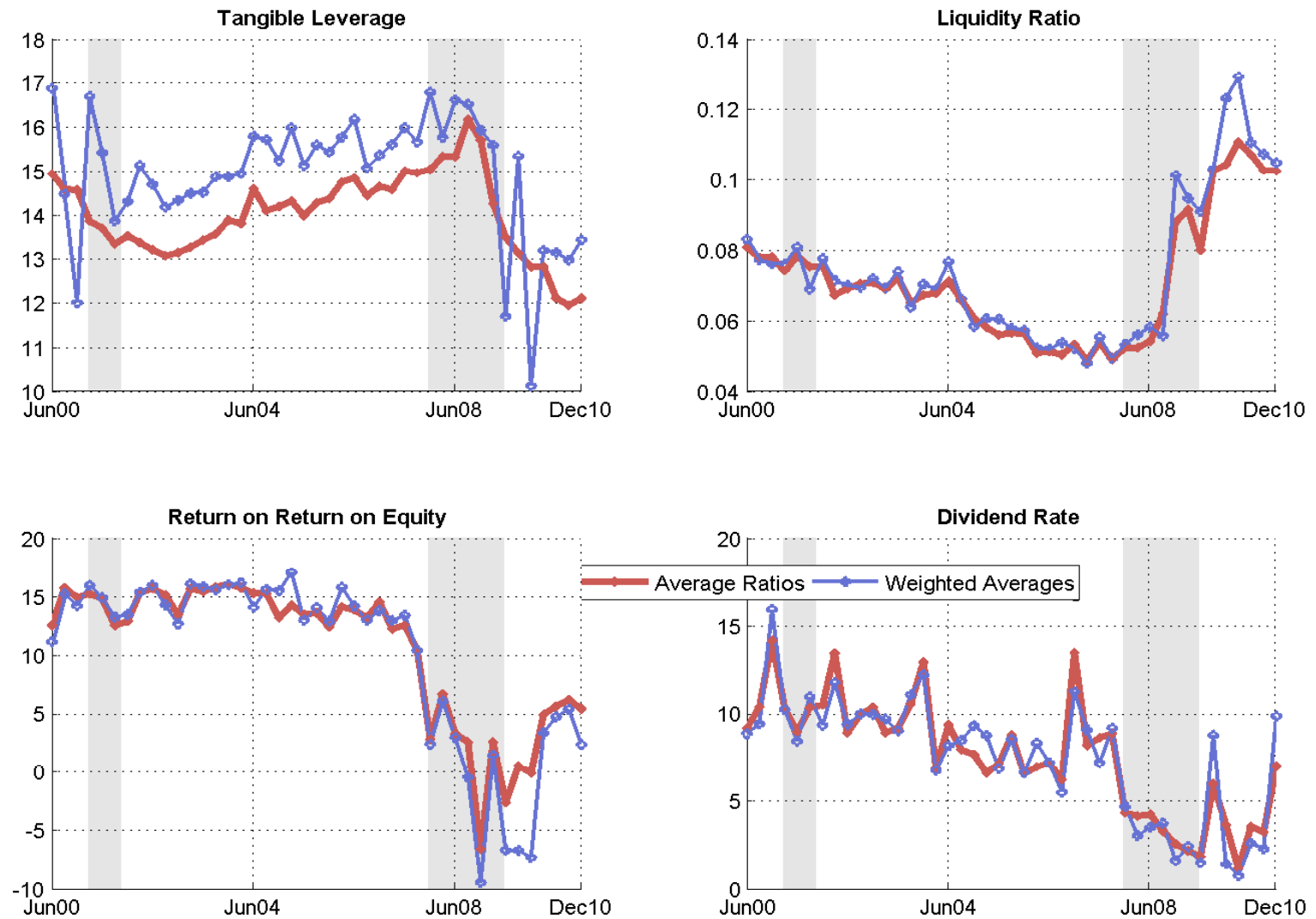


Figure 13: Banking Indicators

8.3 Discussion - Shocks during US Financial Crises

We discuss which is the most plausible combination of shocks to generate the data patterns we observed in the data? The shocks that we have studied so far can be broken up into three categories. The first two shocks, equity losses and increments in capital requirements constraint the entire portfolio of the bank. Since the model is linear at the individual level, either a fall in equity, or a contraction in capital requirements when they are binding, should cause a proportional decline in lending and reserve holdings by banks. Of course, the demand for loans is imperfectly elastic whereas the supply of reserves is perfectly elastic. Thus, either of these shocks causes both a reduction in the supply of credit and the holdings of reserves, but there is a strong substitution away from reserves towards more lending. Overall, the portfolio returns for banks increase, and dividend payments fall short. In practice, these supply explanations are inconsistent with the observed increase in the liquidity ratio of banks.

The shocks associated with higher liquidity costs —higher uncertainty in the withdrawal process, the shut-down of the interbank market or the increase in interest on reserves— have different implications. These shocks cause a substitution away from lending and towards reserve holdings. Overall, these shocks are consistent with the observed decline in lending and the increase in reserve holdings by banks. However, although these shocks may explain the patterns at the beginning of the crises, we know that in practice the Fed did lower policy rates and increased the liquidity of the system. Hence, this second class of shocks cannot explain the aftermath of the crises.

The final shock we studied is a shock to the demand for credit. This story fits decline in profitability by banks and is also consistent with the large holdings of reserves through a substitution effects. These short run effects are summarized by the arrows in the table below.

Our main message is the following. There was an initial contraction in the credit supply by banks that had more to do with uncertainty in the interbank market than with equity losses during the crisis. In practice, this uncertainty may have had to do with risks associated with equity losses, something that our model is silent about, but these operated indirectly through the costs of potential illiquidity. Several quarters after the shock was gone, the patterns observed in the banking sector reflect something that seems closer to be loan demand driven.

9 Conclusions

Modern monetary-macro models have developed independently from banking models. In particular, there is no modern macroeconomic framework to study the implementation of monetary policy through the banking system.³⁸ Recent crises in the US and Europe, however, have revealed that

³⁸In doing so, the profession has lacked an explicit modeling of monetary policy through the financial system, and for good reasons. For a long time it did not seem to make much difference, monetary policy seemed to be carried with ease. In fact, the banking industry showed very stable indicators for equity growth, leverage, dividends and interest premia. Thus, the financial sector can appeared irrelevant for monetary policy of a long stretch of time.

Table 1: Summary of Effects on Impact

	Loans	Cash	Div.	Equity
Equity Loss	↓	↓	↓	↓
Capital Requirement	↓	↓	↓	↑
Uncertainty	↓	↑	↓	↑
Credit Demand	↓	↑	↑	↓
Interest on Reserves	↓	↑	↓	↑
Data	↓	↑	↓	↓

this is lack is a deficiency of modern theory. A model that allows to study the monetary policy and its implementation through banks provides a tool to address many issues that emerge in current policy debates.

This paper attempts to fill that gap. The model studies the implementation of monetary policy through the liquidity management of banks. We then employ the model to understand the effects of various shocks to the banking system. As an application, we argued that a combination of an interbank market freeze first, and a later decline in the demand for loans seems the most plausible story to explain the increase in the holding of reserves and the decline in lending post 2008. We argue that this combination of shocks is suggestive of phenomenon where an initial contraction in the supply of loans engenders a subsequent contraction in the effective loans demand.

We believe the model can be used to answer a number of related questions. Among these, the model can be used to study the Fed's exit strategy from quantitative easing. It can also be used to analyze changes in policy tools. Moreover, the model can be used to estimate the fiscal costs of the Fed's policies of recent years. Similarly, the model can be used to analyze the effects of the policies recently undertaken by the European Central Bank. For open economies, the model can be extended to analyze interventions in the exchange rate market. An extension that breaks aggregation may allow to study the cross-sectional responses of banks depending their liquidity and leverage ratios. We hope that the answers to this questions will provide a guide for policy in practice.

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A Proofs

A.1 Proof of Propositions 1, 2 and 3

This section provides a proof of the optimal policies described in Section 3.4. The proof of Proposition 1 is straight forward by noticing that once E is determined, the banker does not care how he came-up with those resources. The proof of Propositions 2 and 3 is presented jointly and the strategy is guess and verify. Let X be the aggregate state. We guess the following. $V(E; X) = v(X) E^{1-\gamma}$ where $v(X)$ is the slope of the value function, a function of the aggregate state that will be solved for implicitly. Policy functions are given by: $DIV(E; X) = div(X) E$, $\tilde{B}(E; X) = \tilde{b}(E; X) E$, $\tilde{D}(E; X) = \tilde{d}(E; X) E$ and $\tilde{C}(E; X) = \tilde{c}(E; X) E$.

A.1.1 Proof of Proposition 2

Given the conjecture for functional form of the value function, the value function satisfies:

$$V(E; X) = \max_{DIV, \tilde{C}, \tilde{B}, \tilde{D}} U(DIV) + \beta \mathbb{E} \left[v(X') (E')^{1-\gamma} | X \right]$$

$$\text{Budget Constraint:} \quad E = q\tilde{B} + \tilde{C}p + DIV - \frac{\tilde{D}}{R^D}$$

$$\text{Evolution of Equity :} \quad E' = (q'\delta + (1 - \delta)) \tilde{B} + \tilde{C}p' - \tilde{D} - \chi((\rho + \omega'(1 - \rho)) \frac{\tilde{D}}{R^D} - p\tilde{C})$$

$$\text{Capital Requirement :} \quad \frac{\tilde{D}}{R^D} \leq \kappa(\tilde{B}q + \tilde{C}p - \frac{\tilde{D}}{R^D})$$

where the form of the continuation value follows from our guess. We can express all of the constraints in the problem as linear constraints in the ratios of E . Dividing all of the constraints by E , we obtain:

$$\begin{aligned} 1 &= div + q\tilde{b} + p\tilde{c} - \frac{\tilde{d}}{1 + r_d} \\ E'/E &= (q'\delta + 1 - \delta) \tilde{b} + \tilde{c}p' - \tilde{d} - \chi((\rho + \omega'(1 - \rho)) \tilde{D} - p\tilde{C}) \\ \frac{\tilde{d}}{R^D} &\leq \kappa(\tilde{B}q + \tilde{C}p - \frac{\tilde{D}}{R^D}) \end{aligned}$$

where $div = DIV/E$, $\tilde{b} = \tilde{B}/E$, $\tilde{c} = \tilde{C}/E$ and $\tilde{d} = \tilde{D}/E$. Since, E is given at the time of the decisions of B, C, D and DIV , we can express the value function in terms of choice of these ratios.

Substituting the evolution of E' into the objective function, we obtain:

$$\begin{aligned} V(E; X) &= \max_{div, \tilde{c}, \tilde{b}, \tilde{d}} U(div E) + \beta \mathbb{E} \left[v(X') (R(\omega, X, X') E)^{1-\gamma} | X \right] \\ 1 &= div + q\tilde{b} + p\tilde{c} - \tilde{d} \\ \frac{\tilde{d}}{R^D} &\leq \kappa(\tilde{b}q + \tilde{c}p - \frac{\tilde{d}}{R^D}) \end{aligned}$$

where we use the fact that E' can be written as:

$$E' = R(\omega, X, X') E$$

where $R(\omega, X, X')$ is the realized return to the bank's equity and defined by:

$$R(\omega, X, X') \equiv (q(X')\delta + (1-\delta))\tilde{b} + (1+r(X'))\tilde{c} - (1+r_d)\tilde{d} - \chi((\rho + \omega'(1-\rho))(1+r_d)\frac{\tilde{d}}{R^D} - p\tilde{c}).$$

We can do this factorization for E because the evolution of equity on hand is linear in all the term where prices appear. Moreover, it is also linear in the penalty χ also. To see this, observe that $\chi\left((\rho + \omega'(1-\rho))\frac{\tilde{D}}{R^D} - p\tilde{C}\right) = \chi\left((\rho + \omega'(1-\rho))\frac{\tilde{d}E}{R^D} - \tilde{c}E\right)$ by definition of $\{\tilde{d}, \tilde{c}\}$. Since, $E \geq 0$ always, we have that

$$(\rho + \omega'(1-\rho))\frac{\tilde{D}}{R^D} - \tilde{C} \leq 0 \Leftrightarrow \left((\rho + \omega'(1-\rho))\frac{\tilde{d}}{R^D} - \tilde{c} \right) \leq 0.$$

Thus, by definition of χ ,

$$\begin{aligned} \chi((\rho + \omega'(1-\rho))\frac{\tilde{D}}{R^D} - \tilde{C}) &= \begin{cases} E\chi\left((\rho + \omega'(1-\rho))\frac{\tilde{d}}{R^D} - \tilde{c}\right) & \text{if } \left((\rho + \omega'(1-\rho))\frac{\tilde{d}}{R^D} - \tilde{c}\right) \leq 0 \\ E\bar{\chi}\left((\rho + \omega'(1-\rho))\frac{\tilde{d}}{R^D} - \tilde{c}\right) & \text{if } \left((\rho + \omega'(1-\rho))\frac{\tilde{d}}{R^D} - \tilde{c}\right) > 0 \end{cases} \\ &= E\chi\left((\rho + \omega'(1-\rho))\tilde{d} - \tilde{c}\right). \end{aligned}$$

Hence, the evolution of $R(\omega, X, X')$ is a function of the portfolio ratios b, c and d but not of the level of E . With this properties, we can factor out, $E^{1-\gamma}$ from the objective because it is a constant when decisions are made. Thus, the value function may be written as:

$$V(E; X) = E^{1-\gamma} \left[\max_{div, \tilde{c}, \tilde{b}, \tilde{d}} U(div) + \beta \mathbb{E} \left[v(X') R(\omega, X, X')^{1-\gamma} | X \right] \right] \quad (15)$$

$$\begin{aligned} 1 &= div + q\tilde{b} + p\tilde{c} - \tilde{d} \\ \tilde{d} &\leq \kappa(\tilde{B}q + \tilde{C}p - \tilde{d}) \end{aligned} \quad (16)$$

Then, let an arbitrary $\tilde{v}(X)$ be the solution to:

$$\begin{aligned}\tilde{v}(X) &= \max_{div, \tilde{c}, \tilde{b}, \tilde{d}} U(div) + \beta \mathbb{E} \left[\tilde{v}(X') R(\omega, X, X')^{1-\gamma} | X \right] \\ 1 &= div + q\tilde{b} + p\tilde{c} - \frac{\tilde{d}}{R^D} \\ \frac{\tilde{d}}{R^D} &\leq \kappa(\tilde{b}q + \tilde{c}p - \frac{\tilde{d}}{R^D})\end{aligned}$$

We now shows that if $\tilde{v}(X)$ exists, $v(X) = \tilde{v}(X)$ verifies the guess to our Bellman equation. Substituting $v(X)$ for the particular choice of $\tilde{v}(X)$ in (15) allows us to write $V(E; X) = \tilde{v}(X) E^{1-\gamma}$. Note this is true because maximizing over $div, \tilde{c}, \tilde{b}, \tilde{d}$ yields a value of $\tilde{v}(X)$. Since, this also shows that $div, \tilde{c}, \tilde{b}, \tilde{d}$ and independent of E , and $DIV = divE$, $\tilde{B} = \tilde{b}E$, $\tilde{C} = \tilde{c}E$ and $\tilde{D} = \tilde{d}E$.

A.1.2 Proof of Proposition 3

We have from Proposition 2 that

$$\begin{aligned}v(X) &= \max_{\tilde{c}, \tilde{b}, \tilde{d}, div \in \mathbb{R}_+^4} U(div) + \beta \mathbb{E} [v(X') | X] \\ &\mathbb{E}_{\omega'} \left((q'\delta + (1-\delta))\tilde{b} + p'\tilde{c} - \tilde{d} - \chi((\rho + \omega'(1-\rho))\tilde{d}/R^D - p\tilde{c}) \right)^{1-\gamma}\end{aligned}$$

subject to

$$\begin{aligned}1 &= q\tilde{b} + p\tilde{c} + div - \frac{\tilde{d}}{R^D} \\ \frac{\tilde{d}}{R^D} &\leq \kappa \left(q\tilde{b} + \tilde{c}p - \frac{\tilde{d}}{R^D} \right)\end{aligned}$$

Now define:

$$w_b \equiv \frac{\tilde{b}q}{(1-div)}, \quad w_c \equiv \frac{\tilde{c}p}{(1-div)} \quad \text{and} \quad w_d \equiv \frac{\tilde{d}}{R^D(1-div)}.$$

and collecting terms on $1 = q\tilde{b} + p\tilde{c} + div - \frac{\tilde{d}}{R^D}$, we obtain:

$$div + (1-div)(w_b + w_c + w_d) = 1 \Leftrightarrow w_b + w_c - w_d = 1$$

Then using the definition of w_b, w_c, w_d have that $v(X)$

$$\begin{aligned}
v(X) &= \max_{w_b, w_c, w_d} U(div) + \beta \mathbb{E}[v(X') | X] (1 - div)^{1-\gamma} \dots \\
&\quad \mathbb{E}_{\omega'} \left\{ \frac{q'\delta + (1-\delta)}{q} w_b + p'w_c - w_d(R^D) - \chi((\rho + \omega'(1-\rho)) w_d - w_c) \right\}^{1-\gamma} \\
&\quad s.t. \\
w_b + w_c - w_d &= 1 \\
w_d &\leq \kappa(w_b + w_c - w_d)
\end{aligned}$$

Using the definition of returns, we can define portfolio value as:

$$\begin{aligned}
\Omega^*(X) &\equiv \max_{w_b, w_c, w_d} \left\{ \mathbb{E}_{\omega'} (R^B w_b + R^C w_c - w_d R^D - R^X(w_d, w_c))^{1-\gamma} \right\}^{\frac{1}{1-\gamma}} \\
&\quad s.t. \\
w_b + w_c - w_d &= 1 \\
w_d &\leq \kappa(w_b + w_c - w_d)
\end{aligned}$$

Since, the solution to $\Omega(X)$ is the same for any div and using the fact that X is deterministic,

$$v(x) = \max_{w_b, w_c, w_d} U(div) + \beta \mathbb{E}[v(X') | X] (1 - div)^{1-\gamma} \Omega^*(X)^{1-\gamma}$$

which is the formulation in Proposition 3.

For $\gamma \rightarrow 1$, the objective becomes:

$$\Omega(X) = \exp \{ \mathbb{E}_{\omega} [\log(R(\omega, X, X'))] \},$$

and for $\gamma \rightarrow 0$,

$$\Omega(X) = \mathbb{E}_{\omega} [R(\omega, X, X')].$$

Q.E.D

A.2 Proof of Proposition 4

Taking first-order conditions on (5) and using the CRRA functional form for $U(\cdot)$, we obtain:

$$div = (\beta \mathbb{E} v(X') | X)^{-1/\gamma} \Omega^*(X)^{-(1-\gamma)/\gamma} (1 - div) (1 - \gamma)$$

and therefore one obtains:

$$div = \frac{1}{1 + [\beta \mathbb{E} [v(X') | X] (1 - \gamma) \Omega^*(X)]^{1/\gamma}}.$$

Substituting this expression for dividends, one obtains a functional equation for the value function:

$$v(X) = \frac{1/(1-\gamma)}{\left(1 + [\beta \mathbb{E}[v(X')|X] (\Omega^*(X))^{1-\gamma}]^{\frac{1}{\gamma}}\right)^{1-\gamma}} + \beta \mathbb{E}[v(X')|X] (\Omega^*(X))^{1-\gamma} \left[\frac{[\beta \mathbb{E}[v(X')|X] (\Omega^*(X))^{1-\gamma}]^{\frac{1}{\gamma}}}{\left(1 + [\beta \mathbb{E}[v(X')|X] (\Omega^*(X))^{1-\gamma}]^{\frac{1}{\gamma}}\right)^{\frac{1}{\gamma}}} \right]^{1-\gamma}$$

Therefore, we obtain the following functional equation:

$$v(X) = DA \left[1 + (\beta(1-\gamma) \Omega^*(X)^{1-\gamma} \mathbb{E}[v(X')|X])^{\frac{1}{\gamma}} \right]^{\gamma}.$$

We can treat the right hand side of this functional equation as an operator. This operator will be a contraction depending on the values of $(\beta(1-\gamma) \Omega^*(X))^{1-\gamma}$. Theorems in [Alvarez and Stokey \(1998\)](#) guarantee that this operator satisfies the dynamic programming arguments.

In a non-stochastic steady state we obtain:

$$v^{ss} = \frac{1}{1-\gamma} \left(\frac{1}{1 - (\beta \Omega^{*1-\gamma})^{\frac{1}{\gamma}}} \right)^{\gamma}.$$

and

$$div^{ss} = \frac{1}{1 + \left[\beta \left(\frac{1}{1 - (\beta \Omega^{*1-\gamma})^{\frac{1}{\gamma}}} \right)^{\gamma} \Omega^*(X)^{1-\gamma} \right]^{1/\gamma}}.$$

B Evolution of Bank Equity Distribution

Because the economy displays equity growth, equity is unbounded and thus, the support of this measure is the positive real line. Let \mathcal{B} be the Borel σ -algebra on the positive real line. Then, define as $Q_t(e, \mathbf{E})$ as the probability that an individual bank with current equity e transits to the set \mathbf{E} next period. Formally $Q_t : \mathbb{R}_+ \times \mathcal{B} \rightarrow [0, 1]$, and

$$Q(e, \mathbf{E}) = \int_{-1}^1 \mathbb{I} \{e_t(\omega) e \in \mathbf{E}\} F(d\omega)$$

where \mathbb{I} is the indicator function of the event in brackets. Then Q is a transition function and the associated T^* operator for the evolution of bank equity is given by:

$$\Gamma_{t+1}(\mathbf{E}) = \int_0^1 Q(e, \mathbf{E}) \Gamma_{t+1}(e) de.$$

The distribution of equity is fanning out and the operator is unbounded. Gibrat's law shows

that for t large enough Γ_{t+1} is approximated well by a log-normal distribution. Moreover, by introducing more structure into the problem, we could easily obtain a Power law distribution for $\Gamma_{t+1}(\mathbf{E})$. We will use this properties in the calibrated version of the model.

C Data Analysis

D Algorithm

D.1 Steady State

1. Guess prices for loans q and for the probability of a match in the interbank market γ^-, γ^+
2. Solve bank optimization problem 5. Compute value of the bank and dividend payments.
3. Compute associated average equity growth and average surplus in the interbank market.
4. If equity growth equals zero and the conjectured probability of a match in the interbank market is consistent with the average surplus, stop. Otherwise, adjust and continue iterating.

Algorithm to solve transition dynamics in baseline model

D.2 Transitional Dynamics

1. Guess a sequence of loan prices q_t and for the probability of a match in the interbank market γ_t^-, γ_t^+
2. Solve by backward induction banks's dynamic programming problem using 5 for banks' portfolio and 4 for value function and dividend rates
3. Compute growth rate of equity and average surplus in interbank markets
4. Compute price implied by aggregate sequence of loans resulting from (2) and (3), and the probability of a match according to average surpluses computed in (3)
5. If the conjectured price equal effective price from (4) and the average surplus computed in (4) are consistent with the guessed sequences, stop. Otherwise, continue iterating until convergence.

E Derivation of Loan Demand

F Microfoundation for Loan Demand and Deposit Supply

There are multiple ways to introduce a demand for loans and a supply of deposits. Here, the demand for loans emerges from firms who borrow working-capital loans from banks and the supply of deposits from the household's savings decision. With working capital constraints, a low price for loans, q_t , translates immediately into labor market distortions and, therefore, has real output effects. This formulation is borrowed from the classic setup of [Christiano and Eichenbaum \(1992\)](#). To keep the model simple, we deliberately model the real sector so that the loans demand is static—in the sense that it does not depend on future outcomes—and the supply of loans is perfectly elastic.

F.1 Households

Household's Problem. Households obtain utility by consuming and disutility from providing labor. They work during the lending stage and consume during the balancing stage. This distinction is irrelevant for household's but matters for the sequence of events that we describe later. Households have quasi-linear utility in consumption and have a convex cost of providing labor given by $\frac{h_t^{1+\nu}}{1+\nu}$. The only savings instrument available to households are bank deposits and their holding of shares of firms. Households solve the following recursive problem:

$$W(s_t, d_t; X_t) = \max_{\{c_t \geq 0, h_t, d_{t+1} \geq 0\}} c_t - \frac{h_t^{1+\nu}}{1+\nu} + \beta \mathbb{E}[W(s_{t+1}, d_{t+1}; X_{t+1}) | X_t]$$

subject to the budget constraint:

$$d_{t+1} + c_t + p_t^s s_{t+1} = s_t(z_t + p_t^s) + w_t h_t + R_t^D d_t + T_t.$$

Here, β^h is the household's discount factor and ν the inverse of the Frisch elasticity. In the budget constraint, d_t are deposits in banks that earn a real rate of R^D , h_t are hours worked that earn a wage of w_t , and s_t are shares of productive firms. The price of shares is p_t^s and these pay z_t dividends per share. Finally, c_t is the household's consumption and T are lump sum transfers from the government.

The first-order conditions for the household's problem yield the following labor supply:

$$w_t^{\frac{1}{\nu}} = h_t.$$

This supply schedule is static and only a function of real wages. Hence, the total wage income for the household is $w_t^{\frac{1+\nu}{\nu}}$. In turn, substituting the optimality condition in this problem and used the fact that in equilibrium $s_{t+1} = s_t$, we can solve for the optimal policy decisions, $\{c, d\}$, decision independently from the labor choice. The solution is immediate and given by,

$$\{c, d\} = \begin{cases} c_t = w_t^{\frac{1+\nu}{\nu}} + R^D d_t + T; d_{t+1} = 0 & \text{if } R^D < 1/\beta, \\ c_t \in [0, y_t], d_{t+1} = y_t - c_t & \text{if } R^D = 1/\beta, \\ c_t = 0; d' = w_t^{\frac{1+\nu}{\nu}} + R^D d_t + T_t & \text{if } R^D > 1/\beta \end{cases}.$$

These two results imply that workers consume all their cash on hand the period they receive it if the interest is very low and do not save, or carry real balances. If $R^D = 1/\beta^h$, they are indifferent between consuming or savings. Otherwise, they either do not consume or save all their resources. We will consider parameterizations where in equilibrium $R^D = 1/\beta$.

G Firm's Problem - Leeland-Toft Debt

Firms. Firms maximize $\mathbb{E} [\sum_{t=0}^{\infty} m_t z_t]$ where z_t are dividend payouts from the firms and μ_t there stochastic discount factor of the household. Given the linearity of the household's objective, the discount factor is equivalent to $m_t = \beta^t$.

Timing. A continuum of firms of measure one are created at the lending stage of every period. Firms choose a production scale together with a loan size during their arrival period. In periods after this scale choice is decided, firms produces, payback loans to banks and the residual is paid in dividends.

Production Technology. A firm created in period t uses labor h_t , to produce output according to $f_t(h_t) \equiv A_t h_t^{1-\alpha}$. The scale of production is decided during the lending stage of the period when the firm is created. Although the scale of production is determined immediately at the time of creation, output takes time to be realized. In particular, the firm produces $\delta^s (1 - \delta) f_t(h_t)$ of its output during the s -th balancing stage after its scale was decided.

Labor is also employed when the firm is created, and workers required to be paid at that moment.³⁹ Since firms do not posses the cash-flow to pay their workers —no equity injections are possible— the firm needs to borrow from banks to finance the payroll. Firms issue liabilities to the banking sector —loans— by, l_t , in exchange for deposits —bank liabilities—, $q_t l_t$, that the firm can use immediately to pay workers. The repayment of those loans occurs over time. In particular, the firm repays $\delta^s (1 - \delta) l_t$ during the s -th lending stage after the loan was made. Notice that the repayment rate δ coincides exactly with the δ rate of sales. This delivers a problem for firms similar to the one in [Christiano and Eichenbaum \(1992\)](#) with the difference in the maturity. Taking as given wagesm a labor tax τ^l , and the loan prices q_t , the problem of the firm created during the period t is:

³⁹This constraint emerges if it is possible that the firm renagues on this promise and defaults on its payroll [[Bigio 2012](#)]. This enforcement problem is not present between banks and firms or households and banks. If banks can fully enforce their loan contracts, the lack of commitment between firms and workers induces a role for bank debt. @JB: I DON'T FOLLOW THIS LAST COMMENT

$$\max_{\{h_t, l_t\}} \sum_{s=1}^{\infty} \beta^{s-1} z_{t+s-1}$$

subject to:

$$z_{t+s-1} = \delta^s (1 - \delta) A_t f_t(h_t) - \delta^s (1 - \delta) l_t$$

and

$$(1 + \tau_t^l) w_t h_t = q_t l_t.$$

Substituting z_{t+s-1} into the objective function, and substituting the working capita loan, yields a static maximiation problem for firms:

$$\max_{\{h_t\}} A_t f_t(h_t) - (1 + \tau_t^l) w_t h_t / q_t$$

Taking first-order conditions and substituting that $w_t h_t = h_t^{\frac{1+\nu}{\nu}}$ yields an allocation for labor

$$h_t = \left[\frac{q_t A_t (1 - \alpha)}{(1 + \tau_t^l)} \right]^{\frac{\nu}{(1+\alpha\nu)}}.$$

and a demand for loans for new firms:

$$l_t = \frac{[A_t (1 - \alpha)]^{\frac{1+\nu}{(1+\alpha\nu)}}}{(1 + \tau_t^l)^{\frac{\nu(1-\alpha)}{(1+\alpha\nu)}}} q_t^{\frac{\nu(1-\alpha)}{(1+\alpha\nu)}}.$$

which is the expression in 9, and yields the following proposition:

Proposition 9 *The demand for loans takes the form:*

$$q_t = \Theta_t I_t^\epsilon$$

where

$$\Theta_t = \frac{(1 + \tau_t^l)}{[A_t (1 - \alpha)]^{\frac{1+\nu}{\nu(1-\alpha)}}}.$$

and

$$\epsilon = \frac{(1 + \alpha\nu)}{\nu(1 - \alpha)} = \frac{(1/\nu + \alpha)}{(1 - \alpha)}.$$

For, standard calibrations assume $\alpha = \frac{2}{3}$ and $\nu = \frac{1}{2}$. Thus, $\epsilon = \frac{1+\frac{1}{3}}{\frac{1}{2} \cdot \frac{1}{3}} = \frac{\frac{4}{3}}{\frac{1}{6}} = \frac{8}{\frac{1}{6}} = 8$.