

# International Diversification Revisited

by

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## ABSTRACT

Using country index returns from 8 developed countries and 8 emerging market countries, we re-explore the benefits to international diversification over the past 30 years. To examine various theories in a comparable way, we intentionally limited ourselves to an examination of country index returns and a limited number of types of investments. While it is often difficult to find statistically significant improvements in mean returns, the Sharpe ratios from international diversified investments, especially those hedged against currency depreciation, appear to be quite better than the returns investors can obtain from investing strictly in their local country index.

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# 1 Introduction

On October 11, 2013, Floyd Norris, the eminent chief financial correspondent of *The New York Times*, published an article entitled “A Tale of Two Recessions and World Markets, Turned on Their Heads.” Norris noted that in the five years after the 2001-2002 recession from October 9, 2002 to October 9, 2007, world equity markets other than the United States increased by 201% while the United States returned only 104%. Large emerging market countries like Brazil, Russia, India, and China, nicknamed the BRICs, did even better. Brazil produced a return of 1,166%, India returned 609%, China returned 567%, and Russia returned 417%. The next six years from October 9, 2007 to October 9, 2013 told a completely different story. Because of the global financial crisis and the ensuing great recession, only the United States with a return of 7% and Switzerland with a return of 9% offered a positive return. The BRICs did particularly poorly during this period with China experiencing a return of -32%, India at -33%, Brazil at -35%, and Russia at -41%. Investors in the world market excluding the U.S. lost 22% over the six year period.

The story is similar across sectors in the U.S. market. In the period from October 9, 2002 to October 9, 2007, the Energy sector had the best performance of 242%, while the Consumer Staples sector was the worst, returning only 40%. These performances reversed in the period from October 9, 2007 to October 9, 2013 with Consumer Staples returning the second best performance of 41% and Energy returning only 2%. During the latter period, Consumer Discretionary was the best performing sector with a return of 56%, while Financials were the worst performing sector at -45%.

There are two ways of thinking about these data. The stories could tempt a naive investor to think that it is easy to distinguish winners from losers, especially given the long time periods over which the relative performance occurs. Surely, investors should have known or could have learned that the BRICs would do well after the 2001-2002 recession as the world continued on its path of development and globalization, and couldn't an intelligent investor have understood that the U.S. equity market was the best investment choice after the financial crisis? We argue here that the answers to these questions are no. We think the appropriate interpretation of the data is that investors must internationally diversify their equity portfolios to avoid being trapped in a country that does poorly. Only by diversifying internationally can investors avoid missing out on the winning performances of particular countries that are ex post known to be the winners.

The purpose of this paper is to revisit the basic ideas underlying the practice of international diversification of equity portfolios. We begin with a review of the theory. Next we bring the theory to data to examine theory's prediction in real world.

## **2 50 Years of Research on International Diversification**

Over the last 50 years, many theoretical and empirical papers were written about the gains from international diversification. For our selective literature review, we start from the fundamental reasoning of why international diversification might be beneficial, and we then

focus on recent developments and tests to determine whether international diversification is really beneficial, or how to maximize the benefit.

## 2.1 Sharpe Ratios

Modern portfolio theory starts from the proposition that investors naturally like high returns and dislike volatility of returns because it causes losses. The more variable the portfolio return for a given mean, the greater is the probability of loss and the larger are the losses if they occur. The Sharpe ratio is one summary statistic of the risk-return trade-off inherent in a security or a portfolio of securities. The Sharpe ratio measures the average excess return relative to the volatility of the return:

$$SR_{t+1} = \frac{E(r_{t+1} - r_{f,t})}{\sigma_{r_{t+1}}} \quad (1)$$

where  $r_{t+1}$  is the return on an asset or a portfolio,  $r_{f,t}$  is the risk-free rate, and  $\sigma_{r_{t+1}}$  denotes the volatility of the return. It is natural for investors to choose portfolios with high Sharpe ratios because investors want a high excess return with low volatility.

### 2.1.1 When Does International Diversification Improve the Sharpe Ratio?

Consider an investors in the U.S. as an example. All returns are denominated in dollars and investors are considered to have free access to the short-term government bond, which carries a return of  $r_{f,t}$ .<sup>2</sup> If these investors choose not to diversify internationally, the

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<sup>2</sup>It would be preferable to measure returns in real terms as investors ultimately are concerned with the future purchasing power of their investments, but we choose to measure performance in nominal terms

benchmark for comparison is simply the U.S. MSCI market index, which we denote the local return.

If the U.S. investor does not diversify internationally, his Sharpe ratio is

$$SR_{t+1}^{US} = \frac{E(r_{t+1}^{US} - r_{f,t})}{\sigma_{r_{t+1}^{US}}} \quad (2)$$

where  $r_{t+1}^{US}$  is the return on stocks listed in the U.S. and  $\sigma_{r_{t+1}^{US}}$  is the standard deviation of the return. Let the Sharpe ratio of the equity of foreign country  $j$  that the U.S. investor is considering to add to the portfolio be

$$SR_{t+1}^{For(j)} = \frac{E(r_{t+1}^{For(j)} - r_{f,t})}{\sigma_{r_{t+1}^{For(j)}}} \quad (3)$$

and let the correlation between the U.S. market and the foreign return be  $\rho(j)$ .

From a zero investment in foreign equity, the Sharpe ratio of the U.S. investor increases when the investor adds a little bit of foreign equity exposure if the following condition holds:

$$SR_{t+1}^{For(j)} > \rho(j)SR^{US} \quad (4)$$

The appendix to Chapter 7 of Bekaert and Hodrick (2012) proves this statement formally. The inequality states that the U.S. investor's Sharpe ratio improves when a small amount of the foreign asset is added to the U.S. portfolio if the Sharpe ratio of the new asset is higher than the Sharpe ratio of the U.S. portfolio multiplied by the correlation between the U.S. market and the foreign return. In other words, the lower the correlation

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rather than adjusting for inflation to allow comparisons to other analyses that may be familiar to the reader.

of the foreign asset with the U.S. market, the lower the Sharpe ratio of the foreign market can be for it to become an investment that increases the U.S. investor's Sharpe ratio. It is the relatively low correlations across countries that fundamentally makes the argument for international diversification. Accordingly, for the U.S. investor, we can define the hurdle rate for the expected return on international investment in country  $j$  as

$$HurdleRate^{US}(j) = \rho(j) \frac{r^{US}}{\sigma^{US}} \sigma^{For(j)} + r_f. \quad (5)$$

These hurdle rates are presented below in Table 1.

## 2.2 1960s to 1980s: Mean-Variance Frontier

These diversification ideas have been known since the mid 1960s, consequently, international diversification has been advocated since Grubel (1968) and Solnik (1974). One simple way to understand whether international diversification benefits investors and to measure the degree of improvement is to compare the mean returns, volatilities, and Sharpe ratios of investments in the respective local markets and investments in internationally diversified portfolios.

We consider alternative strategies to characterize possible approaches to international diversification. We start from a naive investor, who simply diversifies by investing equal weights in all country indices available in the data set. We denote this strategy as EW. DeMiguel, Garlappi and Uppal (2007) and Tu and Zhou (2011) discuss the problems of choosing portfolio weights with estimated parameters and note that this  $(1/N)$  naive di-

versification strategy works surprisingly well out-of-sample. We also consider a second passive diversification strategy in which we simply invest in the value weighted portfolios of all country indices available, and we denote this strategy as VW. For this strategy, the investor is simply a passive indexer of the international capital market. Obviously, both the EW and VW strategies do not involve short-selling.

The next approach to international diversification assumes that the investor is a period-by-period, mean-variance maximizer, adopting the mean-variance frontier analysis of Sharpe (1964). This diversification strategy is denoted MV. To be more specific, let the  $N \times 1$  vector  $r_t$  denote the returns in period  $t$  on  $N$  assets. The portfolio return is defined as  $r_{p,t} = w_t' r_t$ , where  $w_t$  is the vector of portfolio weights. Given the sample means of the returns,  $\mu_t$ , and the sample covariance matrix,  $\Sigma_t$  at the end of period  $t$ , the investor chooses portfolio weights to minimize the portfolio's variance,

$$w_t' \Sigma_t w_t, \tag{6}$$

subject to the constraint that defines the desired mean return

$$w_t' \mu_t = E(r_p), \tag{7}$$

and the constraint that the portfolio weights must sum to one,

$$w_t' \mathbf{1} = 1. \tag{8}$$

Short-selling constraints can be imposed by adding constraints requiring that the individual portfolio weights,  $w_{i,t} \geq 0$ , for all  $i$ ,  $i = 1, \dots, N$ . The choices of the weights obviously depend on the means and covariance matrix up to time  $t$ , and so the portfolio return  $r_{p,t}$ ,

relying on  $w_t$ , is considered to be in-sample. Alternatively, we refer to  $r_{p,t+1} = w_t' r_{t+1}$  as out-of-sample, because the weight is computed based on information only up to period  $t$ .

### 2.3 1990s: Black-Litterman Modification to Mean-Variance Frontier

While mean-variance analysis has solid theoretical underpinnings, and its in-sample performance is often excellent, its sequential out-of-sample empirical application is often quite problematic. Black and Litterman (1992) and Broadie (1993) note that two problems plague the sequential mean-variance maximizer. First, estimation of the sample means can be quite imprecise, and second, the sample covariance matrix can be nearly singular. Both problems combine to lead to odd or extreme weights on individual assets.

Because of the limitations of the MV strategy, Black and Litterman (1992) suggest that in practice, one should start from the equilibrium weights (namely the VW weights,  $w_{VW,t}$ ) and then tilt towards one's views of future return realizations. The resulting weights, as shown in Black and Litterman (1992), behave much better than the MV weights. Our fourth diversification strategy is therefore based on the Black and Litterman (1992) reasoning, which we denote the BL weights.

To be more specific, following the CAPM, Black and Litterman (1992) assume that the equity premium,  $\pi_t$ , satisfies the following restriction,

$$\pi_t = \gamma \Sigma_t w_{VW,t}, \tag{9}$$



where  $\gamma$  is the coefficient of relative risk aversion. The expected excess return at time  $t$ ,  $\bar{\pi}_t$ , is defined as

$$\bar{\pi}_t = \pi_t + \epsilon_t^{eq}, \text{ with } \epsilon_t^{eq} \rightarrow N(0, \tau \Sigma_t). \quad (10)$$

That is to say, the unobserved expected excess return should be consistent with market equilibrium from last period,  $w_{VW,t}$ , and it should be recognized to be an estimate that contains noise. The uncertainty around the market's expected excess return is proportional to the sample covariance matrix,  $\Sigma_t$ , estimated from previous  $k$  months. Following Black and Litterman (1992), we choose  $\tau = 0.05$ . For our calculations, we back out different  $\gamma$ 's from the observed data for the local countries. In terms of investor's views, we adopt a simple "subjective" view, meaning that the future return realizations are thought to be similar to the past  $k$ -month realized returns, with the same sample mean,  $\mu_t$ , and the same sample covariance matrix,  $\Sigma_t$ . Intuitively, if one asset outperforms in the previous period, this view assumes it will continue to outperform. From this perspective, our view is similar to that of a momentum trader. The view is incorporated by imposing restrictions in the form of

$$I \cdot r_t = \mu_t + \epsilon_t^{view}, \text{ with } \epsilon_t^{view} \rightarrow N(0, \Sigma_t). \quad (11)$$

To combine the equilibrium weight with the views, which are assumed to be normally distributed, Black and Litterman (1992) compute the conditional means and conditional covariance matrix of future returns, and re-do the mean-variance analysis accordingly. Clearly, the resulting weights are a combination of the equilibrium VW weight and the view derived from past returns.

## 2.4 2000s: The Bayesian Flavor

By incorporating a “view”, the Black and Litterman (1992) approach is similar to a Bayesian analysis. The Bayesian analyses of the mean-variance frontier in Kandel and Stambaugh (1995) and Li, Sarkar, and Wang (2003) apply a similar methodology to measure the benefit of international diversification, with and without short-selling constraints, and with and without emerging market assets.

Li, Sarkar, and Wang (2003) assume that the base assets have a multivariate normal distribution with mean  $\mu$  and covariance matrix  $\Sigma$ . They propose three alternative measures of the benefits to international diversification. The first measure is the improvement on the mean return, given that the new portfolio has the same or lower variance than the benchmark or local portfolio. This measure is defined as

$$\delta = \max_w (w' \mu - w'_m \mu \mid w \in C, w' \Sigma w \leq w'_m \Sigma w_m) \quad (12)$$

where  $w'_m$  is the weight of the benchmark portfolio, and the set  $C$  contains the adding up and non-negativity constraints. If international diversification is beneficial,  $\delta$  should be positive and significant.

The second measure of the benefit to international diversification is based on the reduction of volatility, defined as

$$\phi = \min_w \left( 1 - \sqrt{\frac{w' \Sigma w}{w'_m \Sigma w_m}} \mid w \in C, w' \mu \geq w'_m \mu \right) \quad (13)$$

That is,  $\phi$  measures the largest reduction in volatility possible when keeping the mean return the same as or higher than that of the benchmark portfolio. The greater the reduction of

volatility, the closer  $\phi$  is to 1.

Both of the above measures make use of information on the mean return, which can be difficult to estimate. To overcome this problem, Li, Sarkar, and Wang (2003) also create a third measure of the benefit to international diversification, which does not rely on estimation of the mean  $\mu$ :

$$\psi = \min_w \left( 1 - \sqrt{\frac{w' \Sigma w}{w'_m \Sigma w_m}} \mid w \in C \right). \quad (14)$$

The magnitude of  $\psi$  directly measures the reduction in volatility when a local investor switches to the global minimum variance portfolio with international diversification.

The distributions of all three measures are indirect functions of the mean  $\mu$  and the covariance matrix  $\Sigma$ . Li, Sarkar, and Wang (2003) assume uniform prior distributions for  $\mu$  and  $\Sigma$ . That is,

$$p(\mu, \Sigma) = p(\mu) \cdot p(\Sigma), p(\mu) \propto \text{constant}, p(\Sigma) \propto |\Sigma|^{-(n+1)/2}. \quad (15)$$

The posterior distribution is defined as

$$p(\mu, \Sigma | data) = p(\mu | \Sigma, \hat{\mu}, T) \cdot p(\Sigma | \hat{\Sigma}, T), \quad (16)$$

where  $p(\mu | \Sigma, \hat{\mu}, T)$  is multivariate normal with mean  $\mu$  and covariance matrix  $\Sigma/T$ , and  $p(\Sigma | \hat{\Sigma}, T)$  is an inverted Wishart distribution with scale matrix  $T\hat{\Sigma}$  and degrees of freedom  $T - 1$ . Monte Carlo simulation from the posterior distribution gives values for  $\mu$  and  $\Sigma$  from which the empirical distributions of  $\delta$ ,  $\phi$ , and  $\psi$  can be derived.

## 2.5 Current: Diversification with Characteristics

Rather than calculate the conditional means and conditional covariances of assets as in traditional mean-variance analysis, Brandt, Santa-Clara, and Valkanov (2009) choose portfolio shares directly as functions of a limited number of stock characteristics to maximize the expected utility of the investor.

In this approach, at time  $t$  the investor has  $N_t$  available assets. The portfolio weights are taken to be functions of  $K$  observable asset characteristics given by the vector  $x_{i,t}$ . That is, for a  $K$ -dimension vector of parameters,  $\theta$ , that are to be estimated, the portfolio weights are

$$w_{i,t} = \bar{w}_{i,t} + \frac{1}{N_t} \theta' x_{i,t} \quad (17)$$

where  $\bar{w}_{i,t}$  is the weight associated with a benchmark portfolio. The  $N_t \times K$  matrix of characteristics at time  $t$  is  $x_t$ . Each period, each of the asset characteristics is normalized to have mean 0 and variance 1. Thus, the term  $\frac{1}{N_t} \theta' x_{i,t}$  represents a deviation in the weight given to asset  $i$  from the benchmark weight for that asset, and the chosen weights continue to sum to one. Dividing the characteristics by  $N_t$  allows the number of assets to change over time without changing the aggressiveness of the portfolio allocations.

The objective function of the investor is

$$\max_{\{w_{i,t}\}} E_t [u(r_{p,t+1})] \quad (18)$$

where  $u(\cdot)$  is generally taken to be a member of the constant relative risk aversion (CRRA) class of period utility functions although other function such as maximizing the Sharpe

ratio or utility functions characterized by loss aversion are considered.

To estimate the parameters, Brandt, Santa-Clara, and Valkanov (2009) use  $T$  observations to maximize the investor's average utility as in

$$\max_{\{\theta\}} \frac{1}{T} \sum_{t=0}^{T-1} u \left( \sum_{i=1}^{N_t} \left( \bar{w}_{i,t} + \frac{1}{N_t} \theta' x_{i,t} \right) r_{i,t+1} \right) \quad (19)$$

The first order conditions for this problem are

$$\frac{1}{T} \sum_{t=0}^{T-1} h(r_{t+1}, x_t, \theta) = \frac{1}{T} \sum_{t=0}^{T-1} u'(r_{p,t+1}) \left( \frac{1}{N_t} x_t^\top r_{t+1} \right) = 0 \quad (20)$$

These  $K$  equations define a  $K$ -dimensional vector of functions  $h(r_{t+1}, x_t, \theta)$ , and choosing  $\theta$  sets these equations to zero. Thus, the framework produces sample counterparts of Hansen's (1982) GMM orthogonality conditions. Let  $u''(r_{p,t+1})$  represent the second derivative of the utility function, in which case the asymptotic variance of the parameter estimates is

$$\Sigma_\theta = \frac{1}{T} (G^\top V^{-1} G)^{-1} \quad (21)$$

where

$$G = \frac{1}{T} \sum_{t=0}^{T-1} u''(r_{p,t+1}) \left( \frac{1}{N_t} x_t^\top r_{t+1} \right) \left( \frac{1}{N_t} x_t^\top r_{t+1} \right)', \quad (22)$$

and

$$V = \frac{1}{T} \sum_{t=0}^{T-1} h(r_{t+1}, x_t, \theta) h(r_{t+1}, x_t, \theta)'. \quad (23)$$

It is straightforward to impose positivity constraints, but the weights must be re-normalized because they will no longer sum to one. One simply needs to set the new weights equal to

$$w_{i,t}^+ = \frac{\max(0, w_{i,t})}{\sum_{i=1}^{N_t} \max(0, w_{i,t})}. \quad (24)$$

While this complicates the calculation of the standard errors because the new weight function is not differentiable at 0, bootstrap standard errors are easy to implement.

### 3 Data

To re-examine the benefits of international diversification we use monthly returns for country market indices from January 1986 to July 2013. We focus primarily on eight countries: the G7 countries of Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States, which on average account for 82% of world market capitalization, and Netspar's home country, the Netherlands. In the last section of the paper, we apply the same methodology with eight additional emerging markets countries. We obtain data on return indices from MSCI and other data from DataStream. All of our statistics are presented as annualized values.

Table 1 presents summary statistics for the eight developed countries in which all returns are denominated in dollars. Panel A reports results for the full sample, and Panels B and C report the corresponding statistics for the first and second halves of the sample, respectively. In Panel A, the average country index returns in the first row range from 0.066 for Japan to 0.126 for the Netherlands. The return volatilities are reported in the second row. Italy has the highest return volatility at 0.259, and the U.S. has the lowest return volatility at 0.156. The third row presents the annualized Sharpe ratios, defined in equation (1), which range between 0.102 for Italy and 0.468 for the U.S. The fourth row presents the correlations with the U.S. Japan has the lowest correlation at .395 and Canada has the highest at .775. The

fifth row presents the hurdle rates defined in equation (5). These measure the lowest mean return that a foreign country can offer for it to be worthwhile for a U.S. investor to diversify into that country. Of the seven foreign country indexes, the hurdle rates are lower than the mean returns for four countries, and higher than the mean returns for three countries (Germany, Italy and Japan). For example, the hurdle rate for Italy is 0.100, which is higher than its mean return of 0.090, indicating that investing in the Italian country index would not improve a U.S. investor's Sharpe ratio.

In comparing the means of returns in the first rows of Panels B and C, we see that a substantial reduction in the mean returns in the second half of the sample. Other than Canada, which experienced a fall of only 0.001 in its mean return, all other countries experienced a fall in mean returns of between 0.058 for Germany to 0.129 for the U.S. Volatilities for Italy, Japan, and the UK are lower in the second half of the sample, while volatilities for the other countries are higher. The volatility for the Netherlands increases from 0.158 in the first half of the sample to 0.226 in the second half. All Sharpe ratios, except for Canada, are lower in the second half of the sample. The correlations of returns with the U.S. are also consistently higher in the second half of the sample. The increased correlations are not particularly surprising given the large comovements among country indexes during and after the financial crisis. Based on the hurdle rates, for the first half of the sample, six out of seven country indexes offered attractive opportunities for a U.S. based investor, who would have not found Canada to be attractive, while for the second half of the sample, the hurdle rates for five of the seven country indicate desirable international diversification opportunities. For the second half of the sample, if a U.S. based investor

thought that these sample estimates were the true population values, investing in Italy and Japan would not improve the investor's Sharpe ratio. Of course, these estimates are not the true values, and we must perform statistical analysis to fully address the issue of the desirability of international diversification.

## 4 Empirical Implementation of Diversification Strategies

We now turn to individual country analyses in which all investment returns are denominated in the domestic currencies. For example, the Dutch investor's returns are denominated in euros, and we assume that the Dutch investor has access to the domestic currency short-term government bond, with the risk-free return,  $r_f$ . If the Dutch investor chooses not to diversify internationally, the benchmark return is simply the MSCI market index for the Netherlands, which we refer to as the local investment strategy. To assess whether international diversification benefits investors in each of the countries and to measure the degree of improvement from international diversification, we compare the measures of diversification discussed above. We first present results for the equal-weighted and value-weighted strategies as well as the mean-variance and Black-Litterman strategies in Section 4.1. We report the Bayesian analysis in Section 4.2. The conditional diversification strategies based on characteristics are then discussed in Section 4.3.



## 4.1 The Classic Mean-Variance Analysis and Black-Litterman Approach

This section compares the local benchmark strategy to the passive benchmark strategies labeled EW and VW, as well as the sequential mean-variance optimization (MV), and the Black-Litterman (BL) strategies. For both the MV and BL approaches, we report the statistics of the tangency portfolio, which has the highest Sharpe ratio among all combinations of risky assets. We consider four dimensions of variation for both the MV and BL strategies, either in-sample or out-of-sample analysis, and either with or without short-selling constraints. The short-selling constraints are imposed by requiring all weights to be non-negative.

The in-sample analysis estimates the parameters and performs the diversification analysis in the first subsample, 1986-2009 (288 monthly observations). If the investors knew the parameters, this approach would provide the correct analysis. The out-of-sample analysis uses the estimated parameters from the first subsample and applies them to the second subsample, 2010-2013 (45 monthly observations). If there is deterioration in the performance, we conclude that there is instability in the parameter estimates. We report rolling estimates in later Tables.

Table 2 reports the in-sample and out-of-sample results for those two periods. We compute time-series standard deviations using the method of Newey and West (1987) with three lags. Bold fonts indicate that the difference from the local benchmark is statistically significant at the 5% marginal level of significance. Panel A of Table 2 reports the local

monthly mean excess returns in the first column in annualized terms. For example, for the full sample period, the mean excess return for Canada is 0.047, or 4.7% per annum. Using either the EW or VW strategy to passively diversify internationally, the Canadian investor obtains a lower average return of 0.043 or 0.045, respectively. If the Canadian investor adopts mean-variance optimization (MV), the in-sample average return rises to 0.129, and the bold font indicates that this average return represents a statistically significant improvement over the local benchmark. Using the BL strategy also provides a much higher mean return of 0.124, but it is not higher in a statistically significant way. For France, the U.S. and the Netherlands, we see very similar patterns to the Canadian case. The EW and VW strategies have lower mean excess returns than the local strategy. Except for Germany and the Netherlands, the in-sample MV and BL strategies offer statistically significant improvements in the means relative to the local strategy. For Germany, Italy, Japan, and the U.K., the overall patterns are also similar, except that we find that the internationally diversified EW and MV strategies offer an improvement in the mean.

When we impose short-selling constraints, as in the MV-SS and BL-SS strategies, the in-sample mean returns fall substantially but still exceed the local benchmarks, except for France and the Netherlands. In the case of Japan we see a substantive increase from 0.027 to 0.091, but this is not a statistically significant improvement. Clearly, both the MV and BL strategies seem to be able to significantly increase mean returns, but they do so by taking short positions in countries. Adding short-selling constraints limits their performance. The last four columns report the out-of-sample performances of the strategies. For the MV and BL strategies, only Canada, Japan, and the U.S. have positive mean returns,

and the monthly mean returns for Canada fall to 0.001 and 0.012, respectively. Imposing the short-selling constraints improves the out-of-sample mean returns substantially. Except for Canada, the point estimates of the MV-SS and BL-SS strategies consistently outperform the local benchmarks. For other countries, the out-of-sample MV-SS strategies show improvements in the mean returns of between 0.011 for Canada to 0.084 for Japan. The results for BL-SS are similar. None of these improvements reaches statistical significance.

Panel B reports the time series means of the monthly return variances. One of the basic theoretical advantages of international diversification is that it lowers portfolio volatility, and we do consistently find that using the passive EW and VW strategies generally delivers much smaller return volatilities except for the largest countries. Canada, France, Germany, Italy, and the Netherlands all experience statistically significant reductions in return volatility, while Japan experiences lower volatility that is not statistically significant. The UK and the U.S. experience statistically significant increases in volatility for the EW strategy.

Because of the extreme portfolio positions taken in the MV and BL strategies, all countries experience a statistically significant increase in portfolio volatility if there are no short-selling constraints. For the MV strategy, the U.K. and Italy see volatility rise to 0.527 and 0.749, respectively. With short-selling constraints, the volatilities of the MV-SS and BL-SS strategies are lower than the local volatility for Canada, France, Germany, Italy, and the Netherlands, while volatility is higher for Japan, the UK, and the U.S.

Panel C of Table 2 presents the Sharpe ratios. The local benchmark Sharpe ratios

range from 0.082 for Italy to 0.444 for the U.S. Diversifying with the passive EW or VW strategy actually causes a slight decrease in the Sharpe ratio for the Netherlands and the U.S. For the other countries, though, the Sharpe ratios of the VW strategies are higher. Japan experiences the largest increases from 0.134 for the local strategy to 0.416, similar to the U.S. Sharpe ratio. While all in-sample MV and BL strategies deliver higher Sharpe ratios, this performance does not translate to the out-of-sample analysis due to the negative means observed above. When short-selling constraints are imposed in the out-of-sample analysis labeled MV-SS and BL-SS, the Sharpe ratios are higher for all countries ranging from 0.441 to 0.637.

Panel D reports and certainty equivalence returns ( $CEQ$ ), defined as

$$CEQ = \mu_p - \gamma\sigma_p^2, \tag{25}$$

where  $\mu_p$  and  $\sigma_p^2$  are the sample mean and sample variance of the portfolio, and  $\gamma$  is the risk-aversion coefficient computed in the Black and Litterman (1992) approach. The  $CEQs$  measure the return that an investor would demand for sure as an alternative to investing in the mean-variance payoff offered by the particular strategies. The results are quite similar to those in Panel A. For some countries like Japan and Italy, international diversification delivers far better  $CEQs$ , especially in-sample, than the local alternative, while for other countries, the results are more mixed. In addition, only a few of the international diversification strategies offer a statistically significant difference from the local benchmark.

### 4.1.1 A Rolling Sample

While the previous results are often how analyses of the benefits of international diversification are presented, the out-of-sample analysis is unrealistic as an investor would not use the first 24 years of data to estimate the parameters and then stick to those parameters for the next 4 years. As a more realistic alternative, we use a 60 month rolling window. Because our first observation is 1986:01, our first 60 month window is 1986:01-1990:12. Our next window is 1986:02-1991:01. The rolling window serves two purposes. First, it allows time-variation in the means and covariances, and second, it provides an adequate number of out-of-sample observations to make reliable statistical inferences. The in-sample analysis now uses the parameters from the past 60 months, and applies it to the investment in the last month. The out-of-sample analysis applies the rolling parameters to the next out-of-sample observation,

Table 3 reports the time-series means, volatilities, Sharpe ratios, and certainty equivalences for each statistic. We also compute the time-series standard deviations using the method of Newey and West (1987) with three lags. Bold fonts again indicate that the difference from the local benchmark is statistically significant at the 5% marginal level of significance.

Panel A of Table 3 reports the time-series means of the monthly excess returns from the rolling window analysis using the different strategies described at the top of each column. To understand the results, consider the example of the Netherlands. If the investor chooses not to diversify internationally, the local country index delivers an average excess return

of 0.073. If the investor chooses to passively diversify internationally, the EW and VW strategies deliver excess returns of 0.056 and 0.059, respectively, which are both lower than the return to the local country index. When we use the in-sample MV and BL strategies, the excess returns jump spectacularly to 1.276 and 0.490, respectively. Those numbers are no doubt too good to be true for two reasons: they are in-sample estimates, which are not directly investable; and they require extensive short-selling of entire country markets, which could possibly now be done in futures markets but would have been unrealistic back to 1986. When we impose short-selling constraints, the in-sample average excess returns for the Netherlands become 0.107 for both the MV-SS and BL-SS strategies. When we take the estimates out-of-sample, the excess returns from using the MV and BL strategies that allow for short selling become 0.035 and 0.302, respectively; but when short-selling constraints are in place in the out-of-sample analysis, the average excess returns for the MV-SS and BL-SS strategies are 0.060 and 0.066, respectively. The absence of bold font on any of these average returns indicates that none of these averages, even the spectacular in-sample results, are not statistically different from the local average return.

The wildly different results of the in-sample versus out-of-sample analysis without short-selling constraints is perhaps best illustrated by the case of the UK. For the in-sample strategies, both the MV and BL strategies do fabulously well, and the average excess returns of 118.9 percent per annum and 50.7 percent per annum, respectively, while when analyzed out-of-sample, the MV strategy delivers an average loss of 67.2 percent per annum. These results are consistent with the problems encountered in using MV analysis as discussed in Black and Litterman (1992) and Broadie (1993). Our findings confirm that MV analysis

can perform quite terribly out-of-sample, either because the estimates of the mean returns are imprecise, or because of the high correlations across countries which induce extreme long and short weights when there are no constraints.

Figure 1 plots the time-series of the weights from the MV and BL strategies for the U.S. investor. Panel A presents the MV weights when there are no short-selling constraints. With eight countries, the equal weights would be 0.125, but in Figure 1, the maximum and minimum of the axis are  $\pm 200$ , which represents 1,600 times the equal weight. While it is difficult to see that the average weight for an individual asset can be easily around  $\pm 1$ , it is quite easy to see a few outrageous spikes up to 150, or down to -100. The BL weights are designed to do better, and Panel B reports the BL weights when there are no short-selling constraints. While we find that the BL weight behave better than the MV weights before 2011, but the BL weights become even worse afterwards. Panels C and D of Figure 1 report weights when we impose short-selling constraints, for the MV-SS and BL-SS strategies, respectively. While restricting the weights to be between 0 and 1 mechanically reduces the volatility in the weights, Figure 1 indicates that it does not induce diversification. It is often the case that the investor plunges into one country for a few months only to exit and plunge into an alternative country.

Returning to the discussion of Table 3, we think that a rational investor, who would like to adopt an investable strategy, would focus on the comparison between the local, EW, VW, and the out-of-sample MV-SS and BL-SS strategies. For an investor in Japan who is considering international diversification, the local excess return is 0.007. The EW and VW

alternatives deliver excess returns ten times that, or 0.074 and 0.073, respectively. The out-of-sample Japanese MV-SS and BL-SS strategies have excess returns of 0.077 and 0.089, respectively. Simply by comparing the mean returns for these strategies, we find that the highest investable excess return for the Netherlands is achieved by staying local, while for Japan it is achieved by using the BL-SS strategy. Obviously, the benefits of international diversification can differ dramatically across countries, but in terms of statistical significance of the differences across the strategies, there are no significant differences between the average returns on the local markets and the international diversification offered by the EW, VW, and out-of-sample MV-SS and BL-SS strategies.

Nevertheless, a comparison based solely on mean excess returns offers only a partial picture of the gains to international diversification. Panel B of Table 3 compares the volatilities of the monthly returns of the different strategies. We begin again with the Netherlands as an example. If Dutch investors stay local, their return volatility is 0.184. If Dutch investors diversify internationally, their volatilities for the EW and VW strategies are 0.158 and 0.154, respectively, and both of these volatilities are significantly less than the local benchmark. The volatilities of the out-of-sample MV-SS and BL-SS strategies are in between the local and VW strategies, and they are not significantly different from the local volatility. For the eight countries, six (five) out of eight have volatilities of the EW (VW) strategies that are significantly lower than the respective local volatilities, while four out of the eight have out-of-sample volatilities for the MV-SS and BL-SS strategies that are significantly lower than local volatilities. While generalizations are difficult, it seems appropriate to conclude that international diversification generally provides a statistically



significantly reduction in volatility.

Panels C and D report Sharpe ratios and certainty equivalence returns ( $CEQ$ ), respectively. The results are quite similar to those in Panel A. For some countries like Japan and Italy, international diversification delivers better Sharpe ratios and higher  $CEQs$ , while for some other countries, it is the opposite. In addition, there are not many statistically significant differences among the investable alternatives.

#### 4.1.2 A Market Timing Analysis

Previous studies on time-varying asset allocation, such as Ang and Bekaert (2002), argue that there could be contagion in global financial markets during crises, which diminishes the benefits of international diversification. While estimation of a regime switching model as in Ang and Bekaert (2002) would be interesting, we instead adopt a simple market-timing technique to potentially lessen the impact of market crashes on international diversification. Each month  $t$ , we compare the average returns of the eight country indices for the last month with the domestic currency risk-free interest rates. If the average return of the stock indices at time  $t - 1$  is lower than the interest rate for period  $t$ , we assume a crisis has hit, and we flee to safety for month  $t$ , meaning that all equity investments are shifted to the domestic currency interest rate. For the local strategy, we simply compare the local country index with the local interest rate, and we flee to safety if the previous month local country index return is lower than the local interest rate.

We present results for this simple market timing technique in Table 4, which reports the

time-series means, volatilities and Sharpe ratios of excess returns by country of investor in Panels A to C. The overall impression is that timing seems to be important for volatility reduction rather than improving mean returns. Again, take the Netherlands as an example. When there is market timing, the local average excess return decreases from 0.073 to 0.054, and a similar pattern is observed for both EW and VW. The impact of timing on the performance of the MV and BL strategies is minimal. For instance, the excess return on the out-of-sample MV-SS strategy is 0.060 without market timing, and it is 0.053 with market timing. In terms of volatility, though, the local volatility decreases from 0.184 with no market timing to 0.127 with market timing. The volatility drops even further for the EW and VW strategies, but they are not significantly lower than the volatility of the local strategy after market timing. With market timing inducing more of a fall in volatility than in mean, we find in Panel C that the Sharpe ratios increase for almost all strategies.

To better illustrate the magnitudes of the differences among strategies, Figure 2 reports the cumulative returns of different strategies for the U.S. and Japan. Suppose the investor has 1 unit of domestic currency at the beginning of 1991. The plots show how much the money grows till 2013:09. Panels A and B provide cumulative returns without market timing and with market timing, respectively, for a U.S. investor. With no-timing over the 23 years, \$1 grows to \$7.26 if the investor stays local, and it grows to \$6.55 if the investor uses the MV-SS strategy. Those are the top 2 lines in Panel A. Notice, though, that there are big drops in the time-series in 2002 and 2008. In Panel B, when timing is in place, the path becomes smoother, and there are no big drops over the 23 years, and the BL-SS strategy has the best performance. Panel C and D show similar patterns, except that for

Japan, staying local is the worst choice.

## 4.2 Efficiency Gains: A Bayesian Approach

This section investigates the Bayesian analysis following the approach of Li, Sarkar, and Wang (2003). We compute three efficiency gain measures using the eight country return indices over 1986-2013. The results are reported in Table 5. Notice that all optimizations to compute the three measures are conducted in-sample.

Panel A of Table 5 presents results for the three statistics when there are no short-selling constraints. The first statistic,  $\delta$ , is defined in equation (12) and captures the maximum improvement in mean return while controlling for the variance of the return. Panel A reports the mean of  $\delta$ , and the 5<sup>th</sup> and 95<sup>th</sup> percentiles of the distributions using 1000 Monte Carlo simulations. Short-selling constraints are not imposed. For the eight countries, the mean improvement ranges between 0.023 for the U.S. and 0.114 for Italy. These are statistically significant improvements in the means as the 5<sup>th</sup> percentiles of the distributions are all greater than zero. The analysis supports the findings above that international diversification would have been more beneficial for all countries but more so for Germany, Italy, and Japan than for the UK and the U.S.

The second statistic,  $\phi$ , defined in equation (13), measures the reduction in volatility compared to the local country index, while controlling for the mean. For instance, the mean of  $\phi$  is 0.176 for Canada, which indicates that the new optimal portfolio volatility is only about 68% of the local country index volatility ( $1 - \sqrt{0.68} = 0.176$ ). The highest

improvement is again for Italy, and the lowest is for the U.S., but the 5<sup>th</sup> percentile for each country is very close to zero or positive indicating that all countries would benefit from international diversification.

The last statistic,  $\psi$ , defined in equation (14), measures how the global minimum variance portfolio improves relative to the variance of the local country index return. Given that there is no control for the mean return, the differences across countries are slightly smaller, while the highest reduction is again obtained for Italy, and the smallest reduction is obtained for the U.S. From the results in Panel A, the improvements offered by international diversification over the local country indexes in terms of both mean and variance are sizable and significantly different from zero.

In Panel B, we impose short-selling constraints, and we examine whether doing so eliminates the diversification benefits found in Panel A. Starting from  $\delta$ , the means range between 0.010 for the U.S. and 0.077 for Japan, which are smaller in magnitude than those in Panel A, but they are still substantial. For the 5<sup>th</sup> percentile, we see slightly negative numbers in four out of the eight countries, while the other countries have marginally positive efficiency gains. For the improvement in volatility measured by  $\phi$ , the mean reduction is between 0.035 (U.S.) and 0.304 (Italy), indicating that volatility is smaller by 3.47% and 30.43% of the volatilities of the local country indexes. The short-selling constraints have less impact on  $\psi$ , and the results are more similar to those in Panel A. Basically, imposing short-selling constraints cuts the international diversification benefit by about half, but the benefit itself is still mostly positive and substantial.

The results in Panels A and B are based on standard Bayesian analysis, as in Li, Sarkar, and Wang (2003). In Panels C and D, we present summary statistics from the empirical distribution from the time-series of different measures using the 60-month rolling window. Results using the full sample are quite similar to those using 60 month rolling windows and are thus not reported. The numbers in Panel C and D are mostly double the magnitude of those in Panel A and B, but the general patterns are the same. International diversification generates substantial benefits in terms of higher returns, and lower variances. The benefits are reduced if we impose short-selling constraints, but even in that case, the magnitudes are still non-negligible.

### **4.3 Diversifying with Characteristics**

This section examines whether the framework of Brandt, Santa-Clara, and Valkanov (2009) that uses characteristics of stocks to form portfolios can be extended to improve the benefits of international diversification across countries. The idea is to start from the VW weights and use characteristics of countries to chose increases or decreases in the portfolio weights. We use the following characteristics: the previous 6 month return (lag Ret, a momentum or reversal effect), the return volatility (VOL, measured as the annualized volatility of daily returns within the previous month), the market capitalization (MV(\$ MIL), a size effect), the market-to-book ratio (MB, a value effect), the dividend yield (DY), the price-earnings ratio (PE), the term spread (TERM, the difference between the yield on a ten-year government bond and the interest rate on the one-month Treasury bill), and a carry trade

indicator (CARRY, the difference between the one-month domestic currency and USD interest rates).

We report summary statistics on these characteristics except for the lagged return in Panel A of Table 6. The average volatility ranges between 0.146 for the U.S. and 0.233 for Italy. The average market capitalization ranges from \$385 billion for the Netherlands to \$8,687 billion of the U.S. The average market-to-book ratio ranges between 1.81 for the Netherlands and 2.71 for the U.S. The average dividend yield ranges between 1.1% for Japan to 3.7% for the UK. The average price-earnings ratio ranges between 14.53 for France to 42.19 for Japan. The average TERM spread ranges between 0.2% for the UK and 1.5% for Canada. The average interest differential relative to the U.S. ranges between -3% for Japan and 2% for the UK. Basically, one can conclude that our sample of eight countries displays considerable diversity in their characteristics.

Panel B of Table 6 reports the parameter estimates based on the sample from 1986 to 2009. The signs of the parameters are generally consistent across countries, but their magnitudes vary and most parameters are not statistically significant. As one might expect, momentum, the dividend yield, the PE ratio, volatility, the term spread and the carry trade indicator all have positive coefficients, indicating that the investor should choose higher weights than the VW weights for countries with those higher characteristics, while both size and market-to-book carry negative coefficients.

Panels C to E report the in-sample and out-of-sample performance of the BSV approach. In sample, the BSV approach generates higher mean returns, higher volatilities,

and higher Sharpe ratios than the local benchmarks. When short-selling constraints are employed in the BSV-SS strategy, the mean return improvement becomes smaller, but the volatilities also become smaller, and the Sharpe ratios remain slightly higher than the local benchmarks. When we move to the out-of-sample analysis, the BSV approach generates negative mean returns, higher volatilities, and negative Sharpe ratios. When we impose short-selling constraints, both the mean returns and the Sharpe ratios are higher than the local benchmarks and the volatilities are lower than the local benchmarks. This is consistent with our earlier findings using mean-variance analysis. That is, in-sample performance for most strategies looks quite good, but such performance does not persist in the out-of-sample analysis in which the parameters are held constant from the first sample. When we impose short-selling constraints, the benefits of international diversification become clearer.

One possible reason for the bad out-of-sample performance of the BSV strategy is that the parameters might be time-varying, in which case the in-sample estimates do not work well out-of-sample. To check this conjecture, we estimate the parameters associated with the characteristics using 60-month rolling windows to allow the parameters to evolve during the sample. The results are reported in Table 7. Panel A presents summary statistics of the parameter estimates based on the time-series of estimates. We report the parameter means and their t-statistics based on the Newey-West (1987) covariance matrix with twelve lags to allow for the substantial serial correlation in the sequence of estimates.

Across the eight countries, we observe some interesting patterns. For instance, the coefficients on the previous 6 month return are negative ranging from -3.80 for the U.S.

case to -19.73 for the Italian case, and most of the t-statistics approach significance at the usual levels. The negative coefficients indicate a return reversal effect rather than a momentum effect. The coefficients on size are always positive, indicating that investors should put higher weights for larger countries, but the t-statistics indicate that the values are not as significant as the lagged return. Notice that these results run counter to the usual size effect in the cross section of stock returns for firms in which it is argued that small firms outperform large firm. The coefficients on the market-to-book ratio are always negative and significant, which is consistent with the value effect in the cross-sectional literature suggesting that investors should allocate a relatively larger share of their wealth to countries with high book-to-market ratios. The price-earnings ratios always has a negative coefficient, and it is significant in seven out of eight countries. The term spread is always positive and mostly significant indicating that upward sloping term structures predict good returns. The coefficients on past volatility and the dividend yield are mostly positive, but the coefficients are only significant in a couple of countries. The coefficients on the carry trade indicator are all negative, except for Italy, and their t-statistics are all less than one in absolute value. The fact that currencies of relatively high interest rate currencies tend to appreciate relative to those of low interest rates is not sufficiently powerful to offset the relatively poor performance of the equity markets when the country's interest rate is high.

Panels B, C, and D of Table 7 report how international diversification using country characteristics differs from the local and VW strategies for the means, volatilities, and Sharpe ratios. We consider in-sample vs. out-of-sample analysis, as well as with and without short-selling constraints. Clearly, the BSV strategy significantly increases volatility,



both in-sample and out-of-sample. The volatilities do become smaller when short-selling constraints are in place. From Panel D of Table 7, the in-sample BSV and BSV-SS strategies both dominate the local benchmark's Sharpe ratio. While for the out-of-sample analysis, the improvement from the BSV strategy for the mean returns is offset by higher volatilities, and the out-of-sample Sharpe ratios are mostly lower than the local benchmark. After imposing short-selling constraints, the BSV-SS strategy's out-of-sample performance is better than the local benchmark in 6 out of 8 countries and significantly so for Japan.

#### 4.4 Currency Hedged Positions

Investing in global capital markets involves foreign exchange risks, and investors can either hedge these risks or not. If investors choose not to hedge, as in the analysis above, any appreciation (depreciation) of the foreign currency enhances (reduces) the local currency return from the foreign investment. The volatility of the rate of change of the exchange rate also increases the volatility of local currency returns.

We now investigate currency-hedged returns. To understand the issues, consider a U.S. investor at time  $t$  who invests \$1 in the German equity market. If the spot exchange rate of  $\$/\text{€}$  is  $S_t$  and the euro-denominated rate of return in the Germany equity market is  $r_{t+1}^{DE}$ , the unhedged dollar return from the \$1 investment is

$$\frac{S_{t+1}(1 + r_{t+1}^{DE})}{S_t} = 1 + s_{t+1} + r_{t+1}^{DE} + s_{t+1}r_{t+1}^{DE}.$$

where  $s_{t+1} = \frac{S_{t+1}}{S_t} - 1$  is the percentage rate of change of the dollar-euro exchange rate, which is positive for an appreciation of the euro and negative for a depreciation of the euro.

The U.S. investor can hedge the \$1 investment using a one-month forward contract with forward exchange rate  $F_t$  by selling  $(1/S_t)$  euros forward for dollars. In this case the dollar return on the forward contract is

$$\frac{1}{S_t} [F_t - S_{t+1}] = fp_t - s_{t+1}$$

where  $fp_t = \frac{F_t - S_t}{S_t}$  represents the forward premium on the euro, if it is positive, and the forward discount on the euro, if it is negative. The hedged position in dollar terms is the sum of the unhedged dollar return in the German equity market and the dollar return on the forward contract:

$$1 + s_{t+1} + r_{t+1}^{DE} + s_{t+1}r_{t+1}^{DE} + fp_t - s_{t+1} = 1 + fp_t + r_{t+1}^{DE} + s_{t+1}r_{t+1}^{DE}$$

The hedged return essentially replaces the realized rate of appreciation of the euro with the forward premium on the euro. Hedging the currency risk reduces volatility because the volatility of the forward premium is substantially less than the volatility of the realized rate of appreciation. If the foreign currency is consistently at a forward premium that exceeds the average actual rate of appreciation, the mean return is also enhanced. Conversely, if the average forward premium is less than the average rate of appreciation of the foreign currency, hedging reduces the average return.

Table 8 reports the results with hedged returns, using the split sample of 1986-2009 (in-sample) and 2010-2013 (out-of-sample), as above. The results using rolling samples are qualitatively similar and are therefore not reported. We only report the local benchmark and the passive diversification strategies over the full sample, as well as the out-of-sample active strategies, MV, BL and BSV.

In Panel A, when the returns are hedged, the mean returns for the passive EW and VW strategies are about the same as the unhedged values from Table 2 for Canada, France, Germany, and the Netherlands. For Italy and the UK, the mean returns are a bit higher, while for Japan and the U.S., the mean returns from the hedged strategies are lower than their unhedged counterparts. These results are consistent with the intuition from the carry trade. Currencies tend to weaken less than the forward premium on foreign currencies would predict. Thus, relatively high interest rate currencies, like Italy and the UK would have benefited from being unhedged, while relatively low interest rate currencies like Japan and the U.S. would have benefited from unhedged international investments. The results are a bit different for the out-of-sample MV-SS and BL-SS strategies. All countries except Japan have higher mean returns when hedging their foreign investments.

From Panel B, the volatilities of the passive diversification strategies are all lower than their unhedged counterparts. With short-selling constraints, the volatilities of the MV-SS and BL-SS strategies are also significantly lower than the unhedged results for all countries. For example, the unhedged volatility of Japan for the MV-SS strategy is 0.252 while it is 0.126 with hedging. Panel C reports the Sharpe ratios, and hedging clearly improves the Sharpe ratios for all the strategies relative to the local benchmark as well as relative to the unhedged counterparts.

The certainty equivalence returns in Panel D are also quite impressive for the hedge investments. The out-of-sample MV-SS and BL-SS strategies all have certainty equivalence returns ranging from 0.083 for the Netherlands to 0.108 for Japan. Clearly, investors would

have to be compensated handsomely with a risk free return in order for them to forego the returns from the hedged strategies.

## 4.5 Emerging Markets

As mentioned in the introduction, both emerging markets and frontier markets became popular targets for international diversification strategies. Due to data limitations, we only consider 8 emerging markets, and the sample is from 1998 to 2013. Since complete data on forward rate and country characteristics are not available, we only examine the basic strategies, EW, VW, MV and BL.

Table 9 reports the summary statistics for the above strategies using the 60-month rolling window. Take the Netherlands for instance. When the emerging markets are in the play, EW returns are higher than the local benchmark over the same sample period, but not for the VW strategy. Meanwhile, the MV and BL strategies work very well in-sample, but much as before, they are much worse out-of-sample. For a typical emerging market like India or China, international diversification seems to lower the mean returns as well as the variances. In the end, the Sharpe ratios in Panel C don't seem to be significantly different one way or another. There appears to be a lot of instability in the coefficients.

## 5 Conclusions

This paper re-explores the benefits to international diversification. We intentionally limited ourselves to an examination of country returns and a limited number of types of investments. While it is often difficult to find statistically significant improvements in mean returns, the Sharpe ratios from international investments especially those hedged against currency depreciation appear to be quite better than investors can do from investing strictly in their local country index.

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Table 1. Summary Statistics

This table reports the Means, the Volatilities, the Sharpe Ratios, the correlations with the US return, and the Hurdle Rates as defined in equation (5), for the country index returns of 8 countries: Canada (CA), France (FR), Germany (GE), Italy (IT), Japan (JP), the United Kingdom (UK), the United States (US), and the Netherlands (NL). Our full sample period is monthly data from 1986 to 2013. Dollar denominated country index returns are from MSCI.

Panel A. Full Sample

	CA	FR	GE	IT	JP	UK	US	NL
Mean	0.113	0.122	0.110	0.090	0.066	0.108	0.110	0.126
Volatility	0.196	0.218	0.237	0.259	0.223	0.180	0.156	0.196
Sharpe Ratio	0.331	0.348	0.250	0.102	0.242	0.277	0.468	0.377
Correl. w. US	0.775	0.678	0.662	0.510	0.395	0.737	1.000	0.741
Hurdle Rate	0.108	0.107	0.111	0.100	0.081	0.100	0.110	0.106

Panel B. First Half Sample, 1986 – 1999

	CA	FR	GE	IT	JP	UK	US	NL
Mean	0.116	0.182	0.141	0.139	0.121	0.167	0.177	0.190
Volatility	0.170	0.210	0.213	0.266	0.262	0.184	0.151	0.158
Sharpe Ratio	0.258	0.533	0.354	0.153	0.375	0.449	0.816	0.781
Correl. w. US	0.753	0.493	0.438	0.291	0.259	0.619	1.000	0.612
Hurdle Rate	0.156	0.135	0.125	0.111	0.103	0.144	0.177	0.128

Panel C. Second Half Sample, 2000 – 2013

	CA	FR	GE	IT	JP	UK	US	NL
Mean	0.115	0.066	0.083	0.037	0.015	0.051	0.048	0.065
Volatility	0.219	0.225	0.259	0.250	0.174	0.174	0.159	0.226
Sharpe Ratio	0.414	0.194	0.181	0.042	0.075	0.110	0.174	0.119
Correl. w. US	0.808	0.838	0.842	0.720	0.604	0.851	1.000	0.836
Hurdle Rate	0.048	0.049	0.050	0.049	0.046	0.047	0.048	0.049



Table 2. Mean-Variance Analysis, Full Sample

This table reports Means, Volatilities, Sharpe Ratios and Certainty Equivalences for international diversification strategies using MSCI country index returns of 8 countries: Canada (CA), France (FR), Germany (GE), Italy (IT), Japan (JP), the United Kingdom (UK), the United States (US), and the Netherlands (NL). The full sample is 1986 to 2013. All strategies use local currency denominated returns. The non-diversified strategy is the local country index (Local). For passive diversification strategies, we consider equal-weighting (EW) and value-weighting (VW) all countries. For active diversification, we consider mean-variance optimization (MV) and Black-Litterman’s modification of MV analysis (BL). We report the tangency portfolio as the optimal portfolio, which provides the highest Sharpe Ratio. For both MV and BL, we estimate “In-Sample” weights on individual asset using data from 1986 to 2009. For the “Out-of-Sample” statistics, we apply the “In-Sample” parameters to data from 2010 to 2013. For cases with short-selling constraints, MV-SS and BL-SS, the minimum weight is non-negative. With these constraints, we report portfolios with the highest Sharpe Ratios. Bold fonts indicate significant differences from the “Local” benchmarks.

Panel A. Time Series Means of Monthly Excess Returns (annualized)

	Local	EW	VW	In-Sample MV	In-Sample BL	In-Sample MV-SS	In-Sample BL-SS	Out-of-Sample MV	Out-of-Sample BL	Out-of-Sample MV-SS	Out-of-Sample BL-SS
CA	0.047	0.043	0.045	<b>0.129</b>	0.124	0.057	0.057	0.001	0.012	0.058	0.058
FR	0.057	0.044	0.047	<b>0.191</b>	<b>0.180</b>	0.055	0.055	-0.061	-0.031	0.080	0.081
DE	0.050	0.053	0.056	0.163	0.157	0.064	0.064	-0.021	0.000	0.081	0.081
IT	0.019	0.031	0.034	<b>0.331</b>	<b>0.285</b>	0.040	0.040	-0.220	-0.139	0.080	0.080
JP	0.027	0.074	0.074	<b>0.178</b>	<b>0.172</b>	0.091	0.091	0.047	0.063	0.111	0.111
UK	0.040	0.041	0.043	<b>0.252</b>	<b>0.231</b>	0.056	0.056	-0.081	-0.045	0.075	0.075
US	0.069	0.066	0.066	<b>0.140</b>	<b>0.136</b>	0.081	0.079	0.049	0.058	0.086	0.092
NL	0.066	0.052	0.055	0.165	0.159	0.063	0.063	-0.024	-0.002	0.081	0.081

Panel B. Time Series Means of Monthly Return Volatilities (annualized)

	Local	EW	VW	In-Sample MV	In-Sample BL	In-Sample MV-SS	In-Sample BL-SS	Out-of-Sample MV	Out-of-Sample BL	Out-of-Sample MV-SS	Out-of-Sample BL-SS
CA	0.155	0.144	<b>0.139</b>	<b>0.253</b>	<b>0.244</b>	0.151	0.151	<b>0.277</b>	<b>0.263</b>	0.132	0.131
FR	0.198	<b>0.163</b>	<b>0.162</b>	<b>0.405</b>	<b>0.383</b>	0.181	0.181	<b>0.418</b>	<b>0.381</b>	0.128	0.127
DE	0.218	<b>0.163</b>	<b>0.162</b>	<b>0.327</b>	<b>0.316</b>	<b>0.182</b>	<b>0.182</b>	<b>0.326</b>	<b>0.304</b>	<b>0.128</b>	<b>0.127</b>

IT	0.233	<b>0.163</b>	<b>0.162</b>	<b>0.749</b>	<b>0.646</b>	<b>0.181</b>	<b>0.181</b>	<b>0.801</b>	<b>0.669</b>	<b>0.129</b>	<b>0.127</b>
JP	0.203	0.193	0.177	<b>0.319</b>	<b>0.309</b>	0.202	0.202	<b>0.384</b>	<b>0.364</b>	0.252	0.250
UK	0.159	<b>0.167</b>	0.159	<b>0.527</b>	<b>0.485</b>	<b>0.186</b>	<b>0.186</b>	<b>0.574</b>	<b>0.516</b>	0.171	0.168
US	0.156	<b>0.171</b>	0.159	<b>0.247</b>	<b>0.240</b>	<b>0.177</b>	<b>0.173</b>	<b>0.297</b>	<b>0.285</b>	<b>0.202</b>	<b>0.196</b>
NL	0.184	<b>0.163</b>	<b>0.162</b>	<b>0.332</b>	<b>0.321</b>	0.182	0.182	<b>0.332</b>	<b>0.309</b>	0.128	0.127

Panel C. Time Series Means of Sharpe Ratios

	Local	EW	VW	In-Sample MV	In-Sample BL	In-Sample MV-SS	In-Sample BL-SS	Out-of-Sample MV	Out-of-Sample BL	Out-of-Sample MV-SS	Out-of-Sample BL-SS
CA	0.303	0.298	0.323	0.510	0.510	0.376	0.377	0.004	0.044	0.439	0.442
FR	0.287	0.271	0.292	0.472	0.471	0.301	0.302	-0.145	-0.080	0.625	0.635
DE	0.230	0.323	0.343	0.498	0.497	0.354	0.354	-0.064	-0.001	0.628	0.637
IT	0.082	0.191	0.211	0.443	0.442	0.218	0.220	-0.275	-0.208	<b>0.619</b>	<b>0.634</b>
JP	0.134	0.383	0.416	0.558	0.558	0.448	0.449	0.121	0.173	0.442	0.445
UK	0.255	0.248	0.268	0.478	0.477	0.300	0.301	-0.141	-0.086	0.441	0.447
US	0.444	0.385	0.418	0.567	0.567	0.459	0.458	<b>0.167</b>	<b>0.204</b>	<b>0.427</b>	<b>0.469</b>
NL	0.361	0.318	0.338	0.495	0.495	0.349	0.349	-0.071	-0.008	0.628	0.637

Panel D. Time Series Means of Monthly Certainty Equivalences (annualized)

	Local	EW	VW	In-Sample MV	In-Sample BL	In-Sample MV-SS	In-Sample BL-SS	Out-of-Sample MV	Out-of-Sample BL	Out-of-Sample MV-SS	Out-of-Sample BL-SS
CA	0.023	0.023	0.026	0.066	0.066	0.035	0.035	-0.074	-0.056	0.041	0.041
FR	0.028	0.025	0.028	0.072	0.074	<b>0.031</b>	0.031	-0.187	-0.136	0.068	0.069
DE	0.025	0.039	0.042	0.107	0.105	0.047	0.047	-0.077	-0.049	0.072	0.072
IT	0.010	0.026	0.030	0.232	0.212	<b>0.034</b>	0.034	-0.333	-0.218	0.077	0.077
JP	0.014	0.062	0.063	0.145	0.141	<b>0.077</b>	<b>0.077</b>	-0.002	0.019	0.090	0.091
UK	0.020	0.019	0.022	0.029	0.042	<b>0.028</b>	<b>0.028</b>	-0.346	-0.259	0.052	0.052
US	0.035	0.024	0.030	<b>0.053</b>	0.054	<b>0.036</b>	<b>0.037</b>	-0.076	-0.058	0.028	0.037
NL	0.033	0.026	0.029	0.056	0.058	0.031	0.031	-0.132	-0.096	0.064	0.065

Table 3. Mean-Variance Analysis, Rolling Sample

This table reports Means, Variances, Sharpe Ratios and Certainty Equivalences for international diversification strategies using MSCI country index returns of 8 countries: Canada (CA), France (FR), Germany (GE), Italy (IT), Japan (JP), the United Kingdom (UK), the United States (US), and the Netherlands (NL). The full sample is 1986 to 2013. All strategies use local currency denominated returns. The non-diversified strategy is the local country index (Local). For passive diversification strategies, we consider equal-weighting (EW) and value-weighting (VW) all countries. For active diversification, we consider mean-variance optimization (MV) and Black-Litterman’s modification of MV analysis (BL). We report the tangency portfolio as the optimal portfolio, which provides the highest Sharpe Ratio. For both MV and BL, we estimate weights on individual asset, using 60-month rolling windows. We report statistics for In-Sample (using the last in-sample month returns) and Out-of-Sample (using the first out-of-sample month returns). For cases with short-selling constraints, MV-SS and BL-SS, the minimum weight is non-negative. With these constraints, we report portfolios with the highest Sharpe Ratios. Bold fonts indicate significant differences from the “Local” benchmarks.

Panel A. Time Series Means of Monthly Excess Returns (annualized)

	Local	EW	VW	In-Sample MV	In-Sample BL	In-Sample MV-SS	In-Sample BL-SS	Out-of-Sample MV	Out-of-Sample BL	Out-of-Sample MV-SS	Out-of-Sample BL-SS
CA	0.061	0.043	0.043	1.426	0.154	0.104	0.103	-10.683	-0.074	0.051	0.050
FR	0.057	0.053	0.055	1.116	0.579	<b>0.107</b>	0.104	0.032	0.396	0.057	0.064
DE	0.064	0.056	0.058	12.308	0.410	0.105	0.106	17.349	0.264	0.060	0.066
IT	0.030	0.043	0.045	1.162	1.339	<b>0.104</b>	0.096	0.022	0.780	0.051	0.051
JP	0.007	0.074	0.073	1.137	2.062	<b>0.134</b>	<b>0.132</b>	0.469	2.851	0.077	0.089
UK	0.044	0.051	0.052	1.189	0.507	<b>0.121</b>	<b>0.124</b>	-0.672	0.472	0.056	0.055
US	0.071	0.057	0.056	<b>0.888</b>	1.716	<b>0.120</b>	<b>0.124</b>	0.175	1.549	0.068	0.065
NL	0.073	0.056	0.059	1.276	0.490	0.107	0.107	0.035	0.302	0.060	0.066

Panel B. Time Series Means of Monthly Return Volatilities (annualized)

	Local	EW	VW	In-Sample MV	In-Sample BL	In-Sample MV-SS	In-Sample BL-SS	Out-of-Sample MV	Out-of-Sample BL	Out-of-Sample MV-SS	Out-of-Sample BL-SS
CA	0.152	0.137	<b>0.127</b>	<b>5.771</b>	<b>0.451</b>	0.136	0.137	<b>50.598</b>	<b>0.436</b>	0.142	0.143
FR	0.185	<b>0.158</b>	<b>0.154</b>	<b>4.286</b>	<b>1.510</b>	0.169	0.171	<b>1.168</b>	<b>1.224</b>	0.172	0.175
DE	0.210	<b>0.158</b>	<b>0.154</b>	<b>52.897</b>	<b>0.716</b>	<b>0.169</b>	<b>0.170</b>	<b>82.439</b>	<b>0.620</b>	<b>0.172</b>	<b>0.174</b>

IT	0.226	<b>0.158</b>	<b>0.153</b>	<b>4.269</b>	<b>5.293</b>	<b>0.168</b>	<b>0.171</b>	<b>1.224</b>	<b>2.743</b>	<b>0.171</b>	<b>0.174</b>
JP	0.190	0.194	0.176	<b>2.750</b>	<b>4.907</b>	0.197	0.199	<b>3.055</b>	<b>11.665</b>	0.199	0.204
UK	0.143	<b>0.163</b>	0.152	<b>2.611</b>	<b>0.970</b>	<b>0.169</b>	<b>0.172</b>	<b>3.241</b>	<b>0.986</b>	<b>0.171</b>	<b>0.174</b>
US	0.149	<b>0.168</b>	0.151	<b>1.191</b>	<b>3.287</b>	<b>0.174</b>	<b>0.173</b>	<b>1.600</b>	<b>6.845</b>	<b>0.177</b>	<b>0.179</b>
NL	0.184	<b>0.158</b>	<b>0.154</b>	<b>4.313</b>	<b>0.940</b>	0.169	0.170	<b>1.203</b>	<b>0.671</b>	0.172	0.175

Panel C. Time Series Means of Sharpe Ratios (annualized)

	Local	EW	VW	In-Sample MV	In-Sample BL	In-Sample MV-SS	In-Sample BL-SS	Out-of-Sample MV	Out-of-Sample BL	Out-of-Sample MV-SS	Out-of-Sample BL-SS
CA	0.405	0.311	0.343	0.247	0.342	<b>0.763</b>	<b>0.753</b>	<b>-0.211</b>	<b>-0.171</b>	0.360	0.349
FR	0.308	0.337	0.361	0.260	0.383	<b>0.633</b>	<b>0.608</b>	0.027	0.323	0.334	0.367
DE	0.305	0.353	0.378	0.233	0.572	<b>0.621</b>	<b>0.628</b>	0.210	0.426	0.350	0.376
IT	0.131	0.273	0.295	0.272	0.253	<b>0.616</b>	<b>0.563</b>	0.018	0.284	0.299	0.293
JP	0.038	0.381	<b>0.413</b>	0.414	0.420	<b>0.679</b>	<b>0.664</b>	0.153	0.244	0.387	0.436
UK	0.311	0.316	0.340	0.455	0.523	<b>0.715</b>	<b>0.722</b>	-0.207	0.479	0.328	0.318
US	0.479	0.337	0.371	0.746	0.522	0.690	0.720	0.110	0.226	0.381	0.364
NL	0.398	0.356	0.380	0.296	0.521	0.635	0.630	0.029	0.451	0.347	0.380

Panel D. Time Series Means of Monthly Certainty Equivalences (annualized)

	Local	EW	VW	In-Sample MV	In-Sample BL	In-Sample MV-SS	In-Sample BL-SS	Out-of-Sample MV	Out-of-Sample BL	Out-of-Sample MV-SS	Out-of-Sample BL-SS
CA	0.039	0.024	0.028	-30.872	-0.043	0.086	0.085	-2493.377	-0.259	0.032	0.030
FR	0.033	0.036	0.039	-11.775	-1.021	<b>0.087</b>	0.084	-0.925	-0.655	0.037	0.043
DE	0.042	0.043	0.046	-1400.146	0.151	0.091	0.092	-3413.282	0.070	0.045	0.050
IT	0.021	0.039	0.041	-1.764	-3.159	<b>0.099</b>	0.092	-0.219	-0.428	0.047	0.046
JP	-0.003	0.063	0.063	-1.095	-5.044	<b>0.122</b>	<b>0.120</b>	-2.286	-37.319	0.065	0.077
UK	0.028	0.030	0.033	-4.257	-0.244	<b>0.098</b>	<b>0.100</b>	-9.058	-0.304	0.033	0.031
US	0.040	0.017	0.024	<b>-1.104</b>	-13.457	<b>0.077</b>	<b>0.082</b>	-3.422	-64.265	0.023	0.020
NL	0.041	0.032	0.036	-16.697	-0.363	0.080	0.079	-1.364	-0.132	0.031	0.037

Table 4. A Simple Timing Strategy, Rolling Sample

This table reports Means, Variances, Sharpe Ratios and Certainty Equivalences for international diversification strategies using MSCI country index returns of 8 countries: Canada (CA), France (FR), Germany (GE), Italy (IT), Japan (JP), the United Kingdom (UK), the United States (US), and the Netherlands (NL). The full sample is 1986 to 2013. All strategies use local currency denominated returns. We adopt a simple timing strategy. For each month, if the average of the 8 country index returns is lower than the local interest rate, then for the next month, all investment goes to the local interest rate. The non-diversified strategy is the local country index (Local). For passive diversification strategies, we consider equal-weighting (EW) and value-weighting (VW) all countries. For active diversification, we consider mean-variance optimization (MV) and Black-Litterman’s modification of MV analysis (BL). We report the tangency portfolio as the optimal portfolio, which provides the highest Sharpe Ratio. For both MV and BL, we estimate weights on individual asset, using 60-month rolling windows. We report statistics for In-Sample (using the last in-sample month returns) and Out-of-Sample (using the first out-of-sample month returns). For cases with short-selling constraints, MV-SS and BL-SS, the minimum weight is non-negative. With these constraints, we report portfolios with the highest Sharpe Ratios. Bold fonts indicate significant differences from the “Local” benchmarks.

Panel A. Time Series Means of Monthly Excess Returns (annualized)

	Local	EW	VW	Out-of-Sample MV	Out-of-Sample BL	Out-of-Sample MV-SS	Out-of-Sample BL-SS
CA	0.055	0.031	0.014	-10.655	-0.072	0.055	0.053
FR	0.064	0.066	0.058	-0.127	0.200	0.052	0.056
DE	0.040	0.070	0.061	17.214	0.173	0.053	0.062
IT	0.011	0.063	0.055	-0.126	0.563	0.045	0.049
JP	0.010	0.058	0.055	0.590	2.814	0.059	0.067
UK	0.029	0.008	0.021	-0.776	0.330	0.020	0.033
US	0.041	0.045	0.032	0.037	1.394	0.049	0.059
NL	0.054	0.070	0.061	-0.123	0.206	0.053	0.062

Panel B. Time Series Means of Monthly Return Volatilities (annualized)

	Local	EW	VW	Out-of-Sample MV	Out-of-Sample BL	Out-of-Sample MV-SS	Out-of-Sample BL-SS
CA	0.108	0.099	0.090	50.597	0.347	0.105	0.106
FR	0.121	0.110	0.111	1.082	1.054	0.135	0.134
DE	0.153	0.111	0.112	82.440	0.519	0.135	0.132
IT	0.151	0.107	0.108	1.108	2.590	0.134	0.132
JP	0.132	0.127	0.118	2.834	11.653	0.138	0.142
UK	0.090	0.118	0.117	3.099	0.885	0.132	0.130
US	0.104	0.111	0.101	1.588	6.805	0.129	0.124
NL	0.127	0.111	0.112	1.135	0.577	0.135	0.132

Panel C. Time Series Means of Sharpe Ratios (annualized)

	Local	EW	VW	Out-of-Sample MV	Out-of-Sample BL	Out-of-Sample MV-SS	Out-of-Sample BL-SS
CA	0.510	0.314	0.158	-0.211	-0.208	0.521	0.494
FR	0.528	0.598	0.518	-0.118	0.189	0.388	0.419
DE	0.264	0.632	0.547	0.209	0.333	0.394	0.466
IT	0.074	0.589	0.510	-0.114	0.217	0.333	0.373
JP	0.076	0.460	0.464	0.208	0.241	0.426	0.469
UK	0.321	0.069	0.182	-0.251	0.374	0.152	0.254
US	0.397	0.404	0.319	0.024	0.205	0.381	0.478
NL	0.421	0.633	0.549	-0.108	0.357	0.395	0.470

Table 5. Efficiency Gain Analysis

This table reports different efficiency gains as in Li, Sarkar and Wang (2003) using international diversification from the local country index. The data are MSCI country index returns of 8 countries: Canada (CA), France (FR), Germany (GE), Italy (IT), Japan (JP), the United Kingdom (UK), the United States (US), and the Netherlands (NL). The full sample is 1986 to 2013. We define  $\delta$  (the improvement from mean return),  $\phi$  (the improvement from return volatility) and  $\psi$  (the improvement using the global minimum portfolio volatility) in equations (6)-(8). Each reports the means and the 5% and 95% values of these measures for the different countries. In Panels A and B, the distribution is based on 1000 Monte Carlo simulations, without and with short-selling constraints. In Panels C and D, the distributions are based on rolling 60-month samples, without and with short-selling constraints. **The return improvement,  $\delta$ , is annualized.**

Panel A. Monte Carlo Simulations, No Short-Selling Constraints

	$\delta$	$\delta$	$\delta$	$\phi$	$\phi$	$\phi$	$\psi$	$\psi$	$\psi$
	mean	5%	95%	mean	5%	95%	mean	5%	95%
CA	0.047	0.017	0.083	0.176	0.106	0.233	0.196	0.153	0.239
FR	0.059	0.019	0.108	0.195	0.103	0.258	0.223	0.179	0.267
DE	0.094	0.036	0.166	0.279	0.202	0.332	0.293	0.248	0.338
IT	0.114	0.046	0.200	0.329	0.249	0.384	0.343	0.296	0.389
JP	0.104	0.049	0.168	0.186	0.144	0.229	0.187	0.146	0.229
UK	0.029	0.010	0.053	0.069	0.032	0.105	0.080	0.051	0.114
US	0.023	0.007	0.047	0.057	0.024	0.092	0.071	0.043	0.102
NL	0.042	0.013	0.080	0.132	0.068	0.192	0.160	0.120	0.204

Panel B. Monte Carlo Simulations, with Short-Selling Constraints

	$\delta$	$\delta$	$\delta$	$\phi$	$\phi$	$\phi$	$\psi$	$\psi$	$\psi$
	mean	5%	95%	mean	5%	95%	mean	5%	95%
CA	0.024	-0.001	0.061	0.147	0.000	0.226	0.188	0.146	0.233
FR	0.022	-0.001	0.059	0.159	0.000	0.253	0.220	0.176	0.263
DE	0.041	0.000	0.086	0.262	0.000	0.328	0.290	0.244	0.334
IT	0.047	0.000	0.106	0.304	0.000	0.381	0.340	0.294	0.386
JP	0.077	0.017	0.143	0.171	0.128	0.215	0.174	0.132	0.216
UK	0.017	0.000	0.043	0.055	0.008	0.092	0.072	0.042	0.104
US	0.010	-0.006	0.029	0.035	0.000	0.073	0.056	0.030	0.085
NL	0.017	-0.002	0.049	0.102	0.000	0.185	0.156	0.117	0.199

Panel C. Rolling Samples, No Short-Selling Constraints

	$\delta$	$\delta$	$\delta$	$\phi$	$\phi$	$\phi$	$\psi$	$\psi$	$\psi$
	mean	5%	95%	mean	5%	95%	mean	5%	95%
CA	0.121	0.020	0.238	0.261	0.079	0.401	0.319	0.146	0.418
FR	0.168	0.060	0.266	0.299	0.164	0.457	0.324	0.240	0.470

DE	0.202	0.077	0.316	0.355	0.206	0.589	0.385	0.229	0.609
IT	0.240	0.082	0.398	0.390	0.198	0.545	0.407	0.203	0.545
JP	0.222	0.071	0.416	0.293	0.120	0.488	0.306	0.157	0.488
UK	0.101	0.019	0.162	0.173	0.028	0.298	0.191	0.035	0.308
US	0.094	0.037	0.166	0.219	0.104	0.397	0.251	0.123	0.398
NL	0.131	0.026	0.259	0.243	0.021	0.527	0.264	0.023	0.534

Panel D. Rolling Samples, with Short-Selling Constraints

	$\delta$	$\delta$	$\delta$	$\phi$	$\phi$	$\phi$	$\psi$	$\psi$	$\psi$
	mean	5%	95%	mean	5%	95%	mean	5%	95%
CA	0.040	-0.001	0.109	0.129	0.000	0.353	0.255	0.097	0.397
FR	0.052	0.002	0.101	0.219	0.039	0.304	0.268	0.206	0.312
DE	0.055	0.011	0.113	0.292	0.080	0.468	0.339	0.220	0.480
IT	0.071	0.000	0.173	0.309	0.000	0.538	0.353	0.194	0.538
JP	0.113	0.014	0.263	0.223	0.007	0.477	0.228	0.009	0.477
UK	0.026	0.000	0.076	0.074	0.000	0.204	0.088	0.004	0.215
US	0.025	-0.004	0.095	0.074	0.000	0.160	0.131	0.028	0.229
NL	0.041	0.000	0.110	0.170	0.000	0.376	0.211	0.007	0.377



Table 6. Diversifying with Characteristics, Full Sample

This table reports summary statistics of Country Characteristics, Parameter Estimates, Means, Variances, and Sharpe Ratios using different international diversification strategies. Our return data are from MSCI, and sample is 1986 to 2013. Panel A reports the summary statistics for the characteristics variables. Panel B reports coefficient estimates as in Brandt, Santa-Clara, and Valkanov (2009) (BSV), using data from 1986 to 2009, which we consider as “In-Sample”. The t-statistics are based on a Newey-West covariance matrix with 3 lags. We then apply the estimated parameters to the “Out-of-Sample” data from 2010-2013. The country characteristics include the return volatility (VOL), computed as annualized volatility of daily index returns within each month, the market cap (MV), the market to book ratio (MB), the dividend yield (DY), the price to earnings ratio (PE), the term spread (TERM) calculated as the 10-year government bond yield in excess of the 1-month interest rate, and the interest rate differential (CARRY) calculated as the local 1-month interest rate minus the US 1-month interest rate from DataStream. For Panels C to E, we compare BSV’s approach with the local country index and the VW international diversification strategy. We report statistics for In-Sample (1986-2009) and Out-of-Sample (2010-2013). We impose short-selling constraints in the BSV-SS analysis.

Panel A. Summary Statistics of Characteristics

	CA	FR	DE	IT	JP	UK	US	NL
VOL	0.164	0.219	0.221	0.233	0.210	0.182	0.146	0.201
MV(\$ MIL)	693,765	945,593	810,477	421,151	3,110,143	1,846,561	8,687,190	385,273
MB	1.85	1.83	1.85	1.66	1.95	2.02	2.71	1.81
DY	0.025	0.031	0.023	0.031	0.011	0.037	0.022	0.034
PE	17.43	14.53	16.03	17.36	42.19	15.26	18.91	15.23
TERM	0.015	0.010	0.011	0.014	0.014	0.002	0.014	0.012
CARRY	0.007	0.006	-0.003	0.019	-0.030	0.020	0.000	-0.002

Panel B. Parameter Estimates, In-Sample, 1986-2009

		lag Ret	lag MV	lag DY	lag PE	lag MB	lag VOL	lag TERM	lag Carry
CA	para.	1.86	-0.31	5.00	3.27	-0.96	1.05	2.93	3.95
	tstat.	0.90	-0.08	1.23	0.72	-0.32	0.59	1.00	1.05
FR	para.	3.26	-0.06	7.38	6.85	-1.79	1.51	6.68	8.51
	tstat.	1.20	-0.01	1.31	1.10	-0.45	0.65	1.58	1.52
DE	para.	4.51	0.51	9.99	9.40	-2.69	2.33	9.38	12.08
	tstat.	1.22	0.08	1.29	1.08	-0.50	0.76	1.63	1.57
IT	para.	10.95	2.96	24.17	26.08	-6.00	6.10	23.96	34.11
	tstat.	1.18	0.17	1.16	1.08	-0.59	0.89	2.02	1.99
JP	para.	5.15	1.04	14.02	9.84	-2.99	4.94	7.71	11.45
	tstat.	0.90	0.10	1.18	0.72	-0.38	1.04	0.90	1.03
UK	para.	1.32	0.12	3.31	1.83	-0.75	0.68	1.72	2.28
	tstat.	0.93	0.05	1.18	0.59	-0.36	0.52	0.85	0.88
US	para.	2.17	-0.57	6.40	4.30	-0.81	1.60	3.15	4.59

	tstat	0.87	-0.13	1.28	0.77	-0.23	0.75	0.88	0.99
NL	para.	2.37	-0.45	5.55	5.12	-1.14	0.91	4.77	6.06
	tstat	1.18	-0.12	1.34	1.12	-0.38	0.53	1.52	1.47

Panel C. Time Series Means of Monthly Excess Returns (annualized)

	Local	VW	In-Sample BSV	In-Sample BSV-SS	Out-of-Sample BSV	Out-of-Sample BSV-SS
CA	0.061	0.043	0.138	0.050	-0.344	0.074
FR	0.057	0.055	<b>0.240</b>	0.058	-0.683	0.096
DE	0.064	0.058	<b>0.333</b>	0.069	-0.988	0.095
IT	0.030	0.045	<b>0.741</b>	0.045	-2.713	0.083
JP	0.007	0.073	<b>0.432</b>	0.090	-1.010	0.108
UK	0.044	0.052	<b>0.125</b>	0.073	-0.148	0.102
US	0.071	0.056	<b>0.164</b>	0.051	-0.417	0.077
NL	0.073	0.059	<b>0.189</b>	0.066	-0.470	0.098

Panel D. Time Series Means of Monthly Return Volatilities (annualized)

	Local	VW	In-Sample BSV	In-Sample BSV-SS	Out-of-Sample BSV	Out-of-Sample BSV-SS
CA	0.152	<b>0.127</b>	0.276	0.162	0.581	0.115
FR	0.185	<b>0.154</b>	0.427	0.185	0.880	0.112
DE	0.210	<b>0.154</b>	0.568	0.187	1.210	0.113
IT	0.226	<b>0.153</b>	1.342	0.188	2.931	0.118
JP	0.190	0.176	0.768	0.207	1.640	0.224
UK	0.143	0.152	0.231	0.178	0.450	0.173
US	0.149	0.151	0.342	0.184	0.685	0.138
NL	0.184	<b>0.154</b>	0.333	0.184	0.648	0.111

Panel E. Time Series Means of Sharpe Ratios

	Local	VW	In-Sample BSV	In-Sample BSV-SS	Out Sample BSV	Out Sample BSV-SS
CA	0.117	0.099	0.144	0.090	-0.171	0.186
FR	0.089	0.104	0.162	0.091	-0.224	0.249
DE	0.088	0.109	0.169	0.106	-0.236	0.242
IT	0.038	0.085	0.159	0.070	-0.267	0.204
JP	0.011	<b>0.119</b>	0.162	0.125	-0.178	0.139
UK	0.090	0.098	0.157	0.118	-0.095	0.170
US	0.138	0.107	0.138	0.080	-0.176	0.161
NL	0.115	0.110	0.164	0.103	-0.209	0.254

Table 7. Diversifying with Characteristics, Rolling Samples

This table reports summary statistics of Country Characteristics, Parameter Estimates, Means, Variances, and Sharpe Ratios using different international diversification strategies and rolling samples. Our return data are from MSCI, and the full sample is 1986 to 2013. Panel A reports the summary statistics for the characteristics variables. Panel B reports coefficient estimate as in Brandt, Santa-Clara, and Valkanov (2009) (BSV), using data from 1986 to 2009, which we consider as “In-Sample”. The t-statistics are based on a Newey-West covariance matrix with 12 lags. We then apply the estimated parameters to the “Out-of-Sample” data from 2010-2013. The country characteristics include the return volatility (VOL), computed as annualized volatility of daily index returns within each month, the market cap (MV), the market to book ratio (MB), the dividend yield (DY), the price to earnings ratio (PE), the term spread (TERM) calculated as the 10-year government bond yield in excess of the 1-month interest rate, and the interest rate differential (CARRY) calculated as the local 1-month interest rate minus the US 1-month interest rate from DataStream. For Panels C to E, we compare BSV’s approach with the local country index and the VW international diversification strategy. We report statistics for In-Sample (1986-2009) and Out-of-Sample (2010-2013). We impose short-selling constraints in the BSV-SS analysis.

Panel A. Parameter Estimates

		lag Ret	lag MV	lag DY	lag PE	lag MB	lag VOL	lag TERM	lag Carry
CA	para.	-6.22	11.84	0.04	-17.59	-16.85	2.50	6.20	-5.04
	tstat	-1.96	1.37	0.01	-2.29	-2.35	1.03	1.87	-0.79
FR	para.	-6.12	15.32	1.21	-17.39	-22.71	1.16	10.40	-1.14
	tstat	-1.54	1.44	0.20	-1.99	-2.44	0.55	2.46	-0.14
DE	para.	-8.74	20.86	1.19	-23.24	-30.11	1.79	14.62	-0.65
	tstat	-1.60	1.46	0.14	-1.97	-2.40	0.61	2.51	-0.06
IT	para.	-19.73	32.82	11.10	-28.92	-44.38	3.21	29.39	5.44
	tstat	-1.78	1.34	0.83	-1.41	-1.91	0.62	2.51	0.26
JP	para.	-14.55	35.02	5.56	-35.82	-44.83	1.68	17.36	-6.30
	tstat	-1.77	1.57	0.47	-1.93	-2.34	0.29	1.75	-0.38
UK	para.	-6.89	13.62	2.68	-17.32	-20.15	2.84	6.52	-5.48
	tstat	-1.92	1.43	0.50	-2.04	-2.42	1.37	1.61	-0.71
US	para.	-3.80	9.67	0.73	-11.55	-12.35	0.78	4.03	-3.36
	tstat	-1.89	1.76	0.23	-2.20	-2.55	0.62	1.68	-0.77
NL	para.	-4.34	11.30	1.33	-12.79	-16.99	0.71	7.47	-1.22
	tstat	-1.48	1.44	0.29	-1.97	-2.45	0.47	2.40	-0.21

Panel B. Time Series Means of Monthly Excess Returns (annualized)

	Local	VW	In-Sample BSV	In-Sample BSV-SS	Out-of-Sample BSV	Out-of-Sample BSV-SS
CA	0.061	0.043	<b>1.896</b>	0.104	0.340	0.061
FR	0.057	0.055	<b>2.337</b>	0.113	0.336	0.070

DE	0.064	0.058	<b>3.114</b>	0.115	0.455	0.072
IT	0.030	0.045	<b>5.399</b>	0.097	0.908	0.048
JP	0.007	0.073	<b>4.496</b>	<b>0.131</b>	1.121	0.094
UK	0.044	0.052	<b>2.132</b>	<b>0.112</b>	0.310	<b>0.068</b>
US	0.071	0.056	<b>1.258</b>	0.120	0.239	0.077
NL	0.073	0.059	<b>1.754</b>	0.117	0.259	0.073

Panel C. Time Series Means of Monthly Return Volatilities (annualized)

	Local	VW	In-Sample BSV	In-Sample BSV-SS	Out-of-Sample BSV	Out-of-Sample BSV-SS
CA	0.152	<b>0.127</b>	<b>1.291</b>	0.142	<b>1.607</b>	0.148
FR	0.185	<b>0.154</b>	<b>1.616</b>	0.166	<b>1.950</b>	0.171
DE	0.210	<b>0.154</b>	<b>2.190</b>	<b>0.167</b>	<b>2.627</b>	<b>0.172</b>
IT	0.226	<b>0.153</b>	<b>4.253</b>	<b>0.168</b>	<b>5.042</b>	<b>0.168</b>
JP	0.190	0.176	<b>3.303</b>	0.201	<b>3.762</b>	0.206
UK	0.143	0.152	<b>1.482</b>	<b>0.170</b>	<b>1.815</b>	<b>0.172</b>
US	0.149	0.151	<b>0.864</b>	<b>0.173</b>	<b>1.045</b>	<b>0.179</b>
NL	0.184	<b>0.154</b>	<b>1.194</b>	0.166	<b>1.450</b>	0.171

Panel D. Time Series Means of Sharpe Ratios (annualized)

	Local	VW	In-Sample BSV	In-Sample BSV-SS	Out Sample BSV	Out Sample BSV-SS
CA	0.405	0.343	<b>1.469</b>	0.731	0.212	0.415
FR	0.308	0.361	<b>1.446</b>	0.681	0.172	0.408
DE	0.305	0.378	<b>1.422</b>	0.686	0.173	0.418
IT	0.131	0.295	<b>1.269</b>	0.576	0.180	0.284
JP	0.038	<b>0.413</b>	<b>1.361</b>	<b>0.655</b>	0.298	<b>0.455</b>
UK	0.311	0.340	<b>1.439</b>	0.661	0.171	0.397
US	0.479	0.371	<b>1.455</b>	0.692	0.228	0.430
NL	0.398	0.380	<b>1.469</b>	0.705	0.179	0.428

Table 8. Mean-Variance Analysis with Hedged Returns, Full Sample

This table reports Means, Volatilities, Sharpe Ratios and Certainty Equivalences for diversification strategies using MSCI country index returns of 8 countries: Canada (CA), France (FR), Germany (GE), Italy (IT), Japan (JP), the United Kingdom (UK), the United States (US), and the Netherlands (NL). The full sample is 1986 to 2013. All strategies fully hedge the foreign currency investment. The non-diversified strategy is the local country index (Local). For passive diversification strategies, we consider equal-weighting (EW) and value-weighting (VW) all countries. For active diversification, we consider mean-variance optimization (MV), Black-Litterman’s modification of MV analysis (BL), and Brandt, Santa-Clara, and Valkanov’s (2009) conditioning with characteristics (BSV). We report the tangency portfolio as the optimal portfolio, which provides the highest Sharpe Ratio. For both MV and BL, we estimate “In-Sample” weights on individual asset using data from 1986 to 2009. For the “Out-of-Sample” statistics, we apply the “In-Sample” parameters to data from 2010 to 2013. Short-selling constraints are indicated with SS. With these constraints, we report portfolios with the highest Sharpe Ratios. Bold fonts indicate significant differences from the “Local” benchmarks.

Panel A. Time Series Means of Monthly Excess Returns (annualized)

	Full Sample Local	Full Sample EW	Full Sample VW	Out-of-Sample MV	Out-of-Sample BL	Out-of-Sample BSV	Out-of-Sample MVSS	Out-of-Sample BLSS	Out-of-Sample BSV-SS
CA	0.047	0.041	0.050	0.140	0.146	-0.027	0.101	0.107	0.118
FR	0.057	0.047	0.054	0.077	0.086	-0.311	0.098	0.103	0.106
DE	0.050	0.051	0.055	0.072	0.082	-0.440	0.094	0.099	0.104
IT	0.019	0.040	0.053	0.083	0.091	-1.513	0.099	0.103	0.110
JP	0.027	0.050	0.057	0.140	0.145	-0.244	0.109	0.114	0.131
UK	0.040	0.047	0.055	0.138	0.144	-0.065	0.107	0.112	0.120
US	0.069	0.047	0.056	0.148	0.154	0.033	0.110	0.115	0.125
NL	0.066	0.050	0.055	0.073	0.082	-0.192	0.094	0.100	0.106

Panel B. Time Series Means of Monthly Return Volatilities (annualized)

	Full Sample Local	Full Sample EW	Full Sample VW	Out-of-Sample MV	Out-of-Sample BL	Out-of-Sample BSV	Out-of-Sample MVSS	Out-of-Sample BLSS	Out-of-Sample BSV-SS
CA	0.155	0.154	<b>0.150</b>	<b>0.197</b>	<b>0.195</b>	-0.066	0.124	0.127	0.893
FR	0.198	<b>0.155</b>	<b>0.149</b>	0.196	0.194	-0.416	0.128	0.130	0.784
DE	0.218	<b>0.155</b>	<b>0.149</b>	0.219	0.215	-0.452	<b>0.127</b>	<b>0.129</b>	0.767

IT	0.233	<b>0.155</b>	<b>0.149</b>	0.178	0.177	-0.571	<b>0.128</b>	<b>0.130</b>	<b>0.797</b>
JP	0.203	0.153	0.147	0.174	0.174	-0.214	<b>0.124</b>	<b>0.126</b>	<b>0.976</b>
UK	0.159	<b>0.154</b>	0.148	<b>0.189</b>	<b>0.187</b>	-0.126	0.128	0.130	0.903
US	0.156	<b>0.155</b>	0.149	0.181	0.181	0.109	0.126	0.129	0.930
NL	0.184	<b>0.155</b>	<b>0.149</b>	<b>0.216</b>	<b>0.213</b>	-0.356	0.127	0.129	0.791

Panel C. Time Series Mean of Sharpe Ratios (annualized)

	Full Sample Local	Full Sample EW	Full Sample VW	Out-of-Sample MV	Out-of-Sample BL	Out-of-Sample BSV	Out-of-Sample MVSS	Out-of-Sample BLSS	Out-of-Sample BSV-SS
CA	0.303	0.267	0.335	0.711	0.749	-0.194	0.818	0.840	0.101
FR	0.287	0.300	0.363	0.391	0.441	-0.717	0.768	0.794	0.092
DE	0.230	0.327	0.369	0.331	0.381	<b>-0.938</b>	0.738	0.771	0.094
IT	0.082	0.258	0.353	0.465	0.513	-2.753	<b>0.771</b>	<b>0.794</b>	<b>0.106</b>
JP	0.134	0.326	<b>0.388</b>	0.803	0.837	-0.672	0.879	0.900	0.125
UK	0.255	0.307	0.368	0.733	0.767	-0.277	0.841	0.861	0.106
US	0.444	0.307	0.377	0.820	0.851	<b>-0.098</b>	0.874	0.895	0.099
NL	0.361	0.324	0.368	0.336	0.387	<b>-0.477</b>	0.741	0.773	0.088

Panel D. Time Series Means of Monthly Certainty Equivalences

	Full Sample Local	Full Sample EW	Full Sample VW	Out-of-Sample MV	Out-of-Sample BL	Out-of-Sample BSV	Out-of-Sample MVSS	Out-of-Sample BLSS	Out-of-Sample BSV-SS
CA	0.023	0.018	0.028	0.102	0.109	-0.020	0.086	0.091	0.007
FR	0.028	0.029	0.038	0.049	0.058	<b>-0.039</b>	0.086	0.091	0.007
DE	0.025	0.038	0.043	0.047	0.058	<b>-0.051</b>	0.085	0.091	0.007
IT	0.010	0.036	0.049	0.077	0.085	<b>-0.169</b>	0.096	0.100	0.009
JP	0.014	0.042	0.050	0.130	0.135	-0.071	0.104	0.108	0.010
UK	0.020	0.028	0.037	0.110	0.116	<b>-0.029</b>	0.094	0.098	0.008
US	0.035	0.013	0.025	0.102	0.107	<b>-0.011</b>	0.088	0.092	0.007
NL	0.033	0.027	0.033	0.027	0.038	<b>-0.025</b>	0.078	0.083	0.007

Table 9. Mean-Variance Analysis, Including Emerging Markets, Rolling Sample

This table reports Means, Volatilities, Sharpe Ratios, and Certainty Equivalences for different diversification strategies. Our return data are from MSCI, and the sample is 1986 to 2013. For each country, we compare different strategies using local currency denominated returns. The non-diversified strategy is the local country index (Local). For passive diversification strategy, we consider equal-weighting (EW) and value-weighting (VW). For active diversifications, we consider mean-variance frontier optimization (MV), Black-Litterman's modification of MV analysis (BL), and Brandt et al conditioning with characteristics (BSV). We report the tangency portfolio as the optimal portfolio, which obtains the highest Sharpe ratio among all combinations of risky assets. For both MV and BL, we estimate weights on individual assets, using 60-month rolling windows. We report statistics for In-Sample (using the last in-sample month returns) and Out-of-Sample (using the first out-of-sample month returns). For cases with short-selling constraints, MV-SS and BL-SS, the minimum weight is non-negative. With the constraints, we report portfolios with the highest Sharpe ratios. Bold fonts indicate significant differences from the "Local" benchmark.

Panel A. Time Series Means of Monthly Excess Returns (annualized)

	Local	EW	VW	In-Sample MV	In-Sample BL	In-Sample MV-SS	In-Sample BL-SS	Out-of-Sample MV	Out-of-Sample BL	Out-of-Sample MV-SS	Out-of-Sample BL-SS
CA	0.074	0.079	0.036	0.417	0.138	0.179	<b>0.188</b>	0.072	0.040	-0.047	0.084
FR	0.064	0.102	0.058	2.792	3.044	<b>0.209</b>	<b>0.219</b>	0.067	0.059	-0.930	0.103
DE	0.104	0.102	0.058	2.792	3.044	0.209	0.219	0.072	0.059	-0.930	0.103
IT	0.012	0.102	0.058	2.792	3.044	<b>0.209</b>	<b>0.219</b>	0.008	0.059	-0.930	0.103
JP	0.064	0.136	0.090	10.739	0.933	<b>0.229</b>	<b>0.244</b>	0.094	0.090	0.455	0.138
UK	0.075	0.122	0.078	0.381	-0.607	<b>0.239</b>	<b>0.243</b>	0.039	0.072	-0.173	0.116
US	0.077	0.130	0.084	0.830	0.643	0.227	<b>0.238</b>	0.071	0.083	-0.465	0.131
NL	0.072	0.102	0.058	2.792	3.044	0.209	<b>0.219</b>	0.039	0.059	-0.930	0.103
BR	0.055	-0.036	-0.076	0.018	0.017	0.055	0.064	0.091	-0.073	-0.058	-0.031
CN	0.145	0.099	0.052	0.704	-1.032	0.184	0.195	0.169	0.051	0.232	0.098
IN	0.147	0.102	0.059	0.584	5.331	0.204	0.213	0.105	0.066	3.173	0.110
KR	0.116	0.102	0.059	1.154	0.063	0.216	0.216	0.071	0.055	0.479	0.098
MX	0.143	0.095	0.051	0.289	0.591	0.189	0.193	0.120	0.054	-0.344	0.098
RU	0.086	0.051	0.008	0.086	0.265	0.156	0.169	0.120	0.008	-0.432	0.052
ZA	0.111	0.093	0.052	0.756	2.368	0.196	0.200	0.046	0.061	-0.685	0.102
TW	0.074	0.118	0.073	1.223	0.333	0.213	<b>0.220</b>	0.038	0.072	-0.050	0.117

Panel B. Time Series Means of Monthly Return Volatilities (annualized)

	Local	EW	VW	In-Sample MV	In-Sample BL	In-Sample MV-SS	In-Sample BL-SS	Out-of-Sample MV	Out-of-Sample BL	Out-of-Sample MV-SS	Out-of-Sample BL-SS
CA	0.140	0.155	<b>0.118</b>	<b>0.827</b>	<b>0.536</b>	<b>0.218</b>	<b>0.211</b>	<b>0.102</b>	<b>0.117</b>	<b>0.280</b>	<b>0.154</b>
FR	0.168	0.169	<b>0.134</b>	<b>8.682</b>	<b>9.483</b>	<b>0.243</b>	<b>0.232</b>	<b>0.106</b>	<b>0.134</b>	<b>3.371</b>	<b>0.169</b>
DE	0.199	<b>0.169</b>	<b>0.134</b>	<b>8.682</b>	<b>9.483</b>	<b>0.243</b>	<b>0.232</b>	<b>0.126</b>	<b>0.134</b>	<b>3.371</b>	<b>0.169</b>
IT	0.201	<b>0.169</b>	<b>0.134</b>	<b>8.682</b>	<b>9.483</b>	<b>0.243</b>	0.232	<b>0.132</b>	<b>0.134</b>	<b>3.371</b>	<b>0.169</b>
JP	0.190	<b>0.240</b>	0.202	<b>32.238</b>	<b>2.251</b>	<b>0.295</b>	<b>0.281</b>	<b>0.132</b>	<b>0.201</b>	<b>2.469</b>	<b>0.240</b>
UK	0.137	<b>0.182</b>	0.145	<b>0.696</b>	<b>2.117</b>	<b>0.242</b>	<b>0.233</b>	<b>0.088</b>	<b>0.146</b>	<b>0.472</b>	<b>0.183</b>
US	0.147	<b>0.213</b>	0.167	<b>1.298</b>	<b>1.854</b>	<b>0.268</b>	<b>0.256</b>	<b>0.100</b>	<b>0.167</b>	<b>1.650</b>	<b>0.213</b>
NL	0.179	0.169	<b>0.134</b>	<b>8.682</b>	<b>9.483</b>	<b>0.243</b>	<b>0.232</b>	<b>0.116</b>	<b>0.134</b>	<b>3.371</b>	<b>0.169</b>
BR	0.223	<b>0.160</b>	<b>0.140</b>	<b>0.058</b>	<b>0.054</b>	0.224	0.220	<b>0.152</b>	<b>0.140</b>	0.189	0.159
CN	0.278	<b>0.212</b>	<b>0.166</b>	<b>0.935</b>	<b>4.591</b>	0.273	0.260	<b>0.201</b>	<b>0.166</b>	0.447	0.212
IN	0.266	<b>0.172</b>	<b>0.134</b>	<b>1.108</b>	<b>12.674</b>	<b>0.230</b>	<b>0.222</b>	<b>0.205</b>	<b>0.135</b>	11.956	<b>0.173</b>
KR	0.212	<b>0.156</b>	<b>0.123</b>	<b>1.435</b>	<b>2.182</b>	0.212	0.207	<b>0.150</b>	<b>0.123</b>	1.268	0.156
MX	0.175	0.162	<b>0.121</b>	<b>0.399</b>	<b>2.843</b>	<b>0.217</b>	<b>0.210</b>	<b>0.135</b>	<b>0.121</b>	<b>1.691</b>	<b>0.162</b>
RU	0.333	<b>0.169</b>	<b>0.134</b>	<b>0.258</b>	<b>0.688</b>	<b>0.260</b>	<b>0.256</b>	<b>0.226</b>	<b>0.134</b>	<b>1.240</b>	<b>0.169</b>
ZA	0.165	0.173	0.158	<b>2.798</b>	<b>5.431</b>	<b>0.230</b>	<b>0.219</b>	<b>0.107</b>	<b>0.155</b>	<b>2.539</b>	<b>0.170</b>
TW	0.212	0.187	<b>0.145</b>	<b>2.020</b>	<b>0.394</b>	<b>0.242</b>	0.232	<b>0.155</b>	<b>0.145</b>	<b>0.494</b>	0.187

Panel C. Time Series Means of Sharpe Ratios (annualized)

	Local	EW	VW	In-Sample MV	In-Sample BL	In-Sample MV-SS	In-Sample BL-SS	Out-of-Sample MV	Out-of-Sample BL	Out-of-Sample MV-SS	Out-of-Sample BL-SS
CA	0.529	0.508	0.305	0.504	0.258	0.823	0.889	0.708	0.347	-0.169	0.545
FR	0.382	0.600	0.436	0.322	0.321	0.862	0.945	0.636	0.441	-0.276	0.608
DE	0.524	0.600	0.436	0.322	0.321	0.862	0.945	0.570	0.441	-0.276	0.608
IT	0.059	0.600	0.436	0.322	0.321	0.862	<b>0.945</b>	0.058	0.441	-0.276	0.608
JP	0.335	0.568	0.445	0.333	0.414	0.777	0.869	0.715	0.448	0.184	0.573
UK	0.547	0.669	0.537	0.548	-0.287	0.986	1.042	<b>0.435</b>	0.492	-0.366	0.636
US	0.526	0.613	0.501	0.639	0.347	0.847	0.928	0.708	0.498	-0.282	0.613



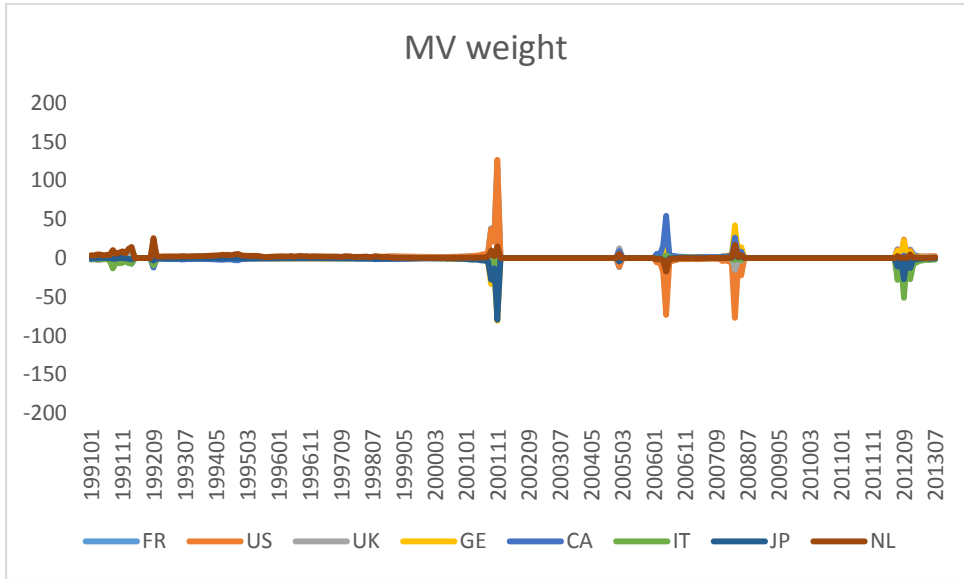
NL	0.402	0.600	0.436	0.322	0.321	0.862	0.945	0.336	0.441	-0.276	0.608
BR	0.248	-0.223	<b>-0.544</b>	0.309	0.309	0.247	0.293	0.596	-0.518	-0.309	<b>-0.197</b>
CN	0.522	0.465	0.313	0.753	-0.225	0.674	0.748	0.839	0.308	0.518	0.463
IN	0.554	0.594	0.436	0.527	0.421	0.888	0.958	0.513	0.489	0.265	0.638
KR	0.546	0.654	0.483	0.804	0.029	1.020	1.045	0.475	0.449	0.378	0.630
MX	0.821	0.585	0.424	0.723	0.208	0.871	0.917	<b>0.884</b>	0.444	-0.203	0.604
RU	0.256	0.301	0.058	0.333	0.385	0.600	0.659	0.531	0.063	-0.348	0.308
ZA	0.673	0.537	0.332	0.270	0.436	0.852	0.915	<b>0.431</b>	0.390	-0.270	0.596
TW	0.348	0.628	0.501	0.606	0.846	0.879	0.951	0.245	0.495	-0.102	0.626

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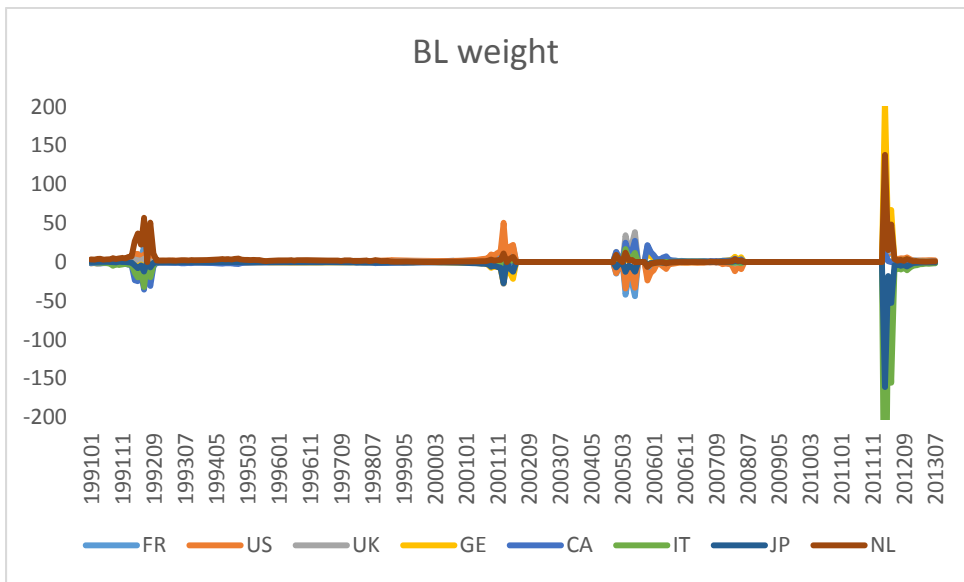
Figure 1. Weights for Rolling Sample

We plot the time-series of portfolio weights on individual assets over 1991-2013 using different active diversification strategies.

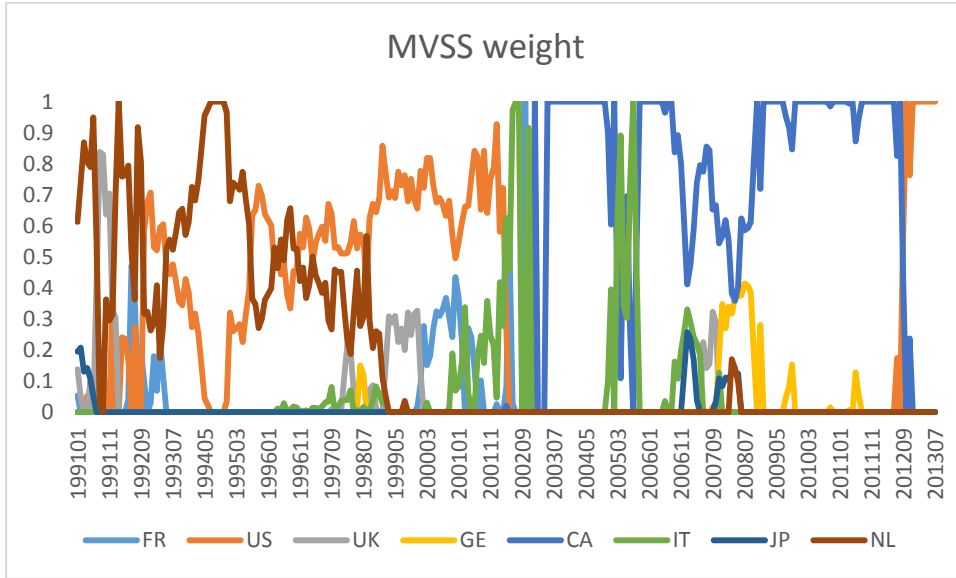
Panel A. US Local Investors, Weights of Tangency Portfolios on Efficient Frontier



Panel B. US Local Investors, Weights of Tangency Portfolios on Efficient Frontier using the Black-Litterman Approach



Panel C. US Local Investors, Weights of Tangency Portfolio on Efficient Frontier with Short Sale Constraints



Panel D. US Local Investors, Weights of Tangency Portfolio on Efficient Frontier, using the Black-Litterman Approach with Short Sale Constraints

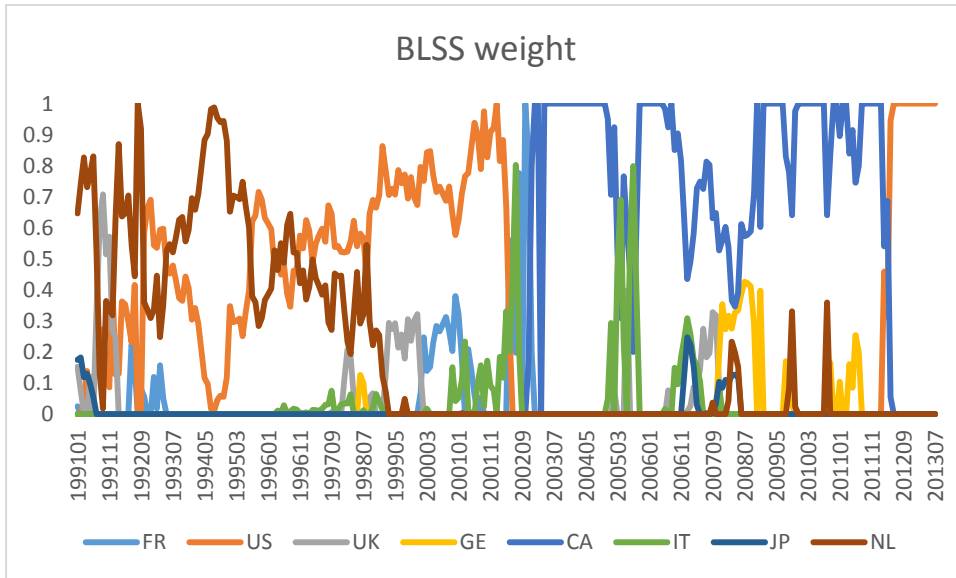
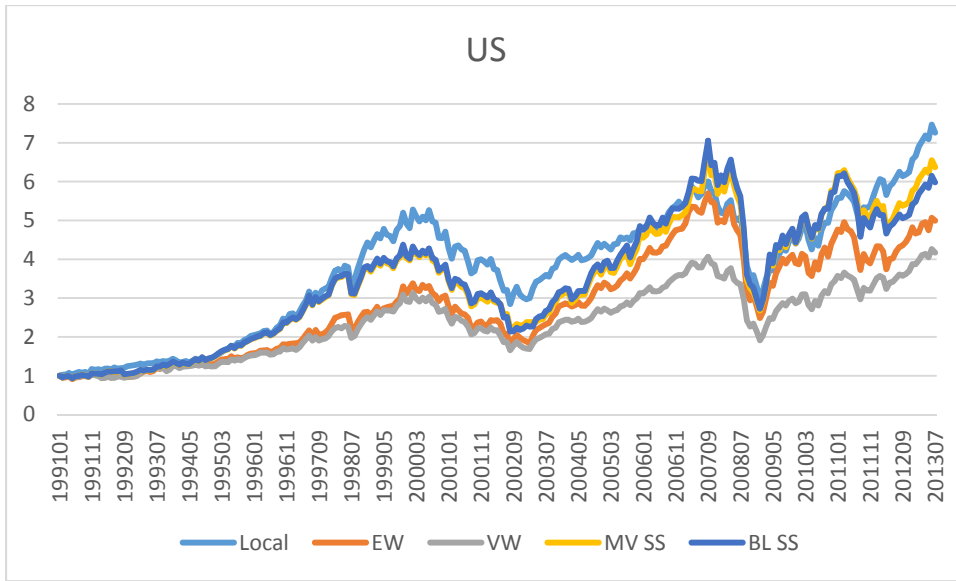
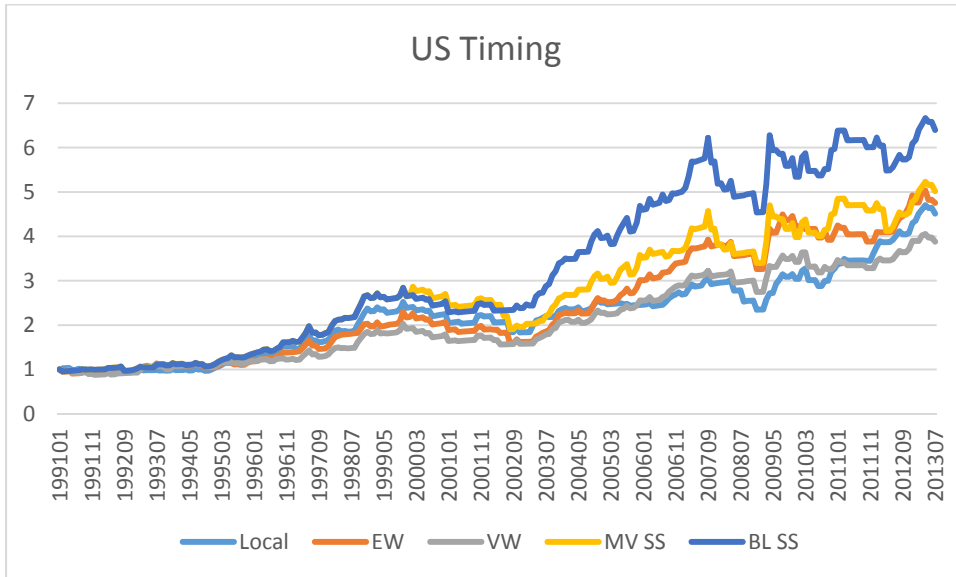


Figure 2. Cumulative Returns for Passive and Active Diversification Strategies

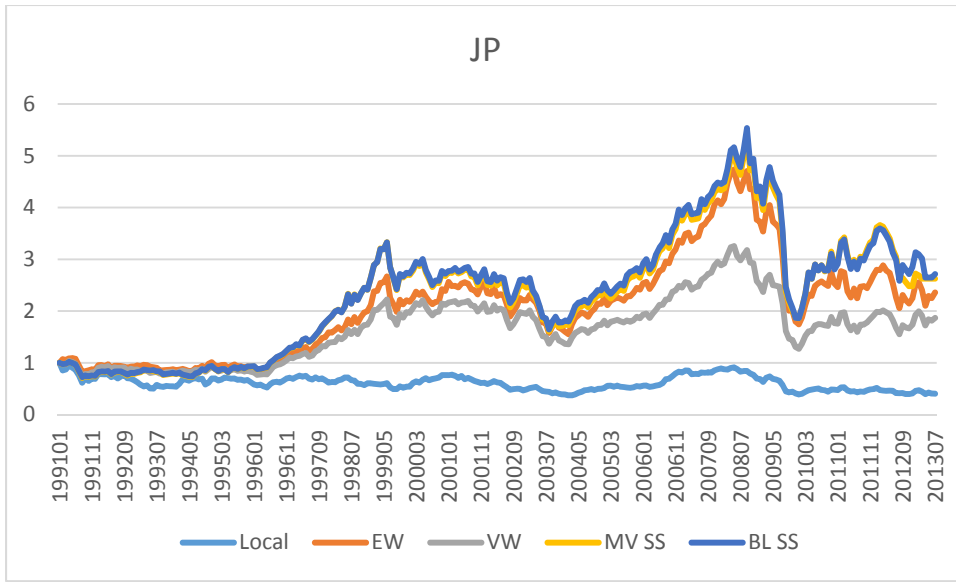
Panel A. US Local Investors, No Timing



Panel B. US local investors, timing



Panel C. JP local investors, no timing



Panel D. JP local investors, timing

