Pricing a Bundle of Products or Services: The Case of Nonprofits

Asim Ansari; S. Siddarth; Charles B. Weinberg

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ASIM ANSARI, S. SIDDARTH, and CHARLES B. WEINBERG*

The authors determine the optimal number of items to be included in a service bundle for a profit-maximizing firm that uses pure components, pure bundling, or mixed bundling strategies. When applied to Venkatesh and Mahajan's (1993) data, the number of events held is shown to have a substantial impact on firm profits. The authors also study the pricing strategies of a nonprofit organization that seeks to maximize usage subject to a nondeficit constraint. Using the same data, the authors show that compared to a profit maximizing firm, a usage-maximizing nonprofit organization (1) charges lower prices, (2) holds more events, and (3) takes fixed costs into account in setting prices. For the distributions considered by Venkatesh and Mahajan (1993), there is a reversal in the order of preferred strategies, with the pure-bundling strategy dominating the single ticket strategy, though mixed bundling is still the most preferred strategy. In addition, the authors find that attendance is *not* maximized by offering the greatest possible number of events. The effects of alternative objective functions, such as surplus maximization, and additional bundling policies (adding bundles consisting of only some of the events scheduled) are also examined.

Pricing a Bundle of Products or Services: The Case of Nonprofits

In a recent study, Venkatesh and Mahajan (V&M) (1993) develop a probabilistic approach to help a seller determine the profit-maximizing prices of a bundle and/or its components under three strategies: pure components (sell only individual items), pure bundling (sell the items only as a package), or mixed bundling (sell the items either individually or as a package). They estimate optimal profits for the three policies and determine the global profit-maximizing strategy.

We extend V&M's results in three ways. First, V&M assume the number of different items to include in the package is known a priori. We propose a model in which the (optimal) number of items to include is determined endogenously. Second, we determine the optimal policy for a nonprofit service organization characterized by an objective function that maximizes usage or number of users (within a defined product-market segment) and is subject to a nondeficit constraint (Weinberg 1980). Third, we consider a broader range of policies for packaging subscriptions and

investigate the effect of a surplus maximization objective function on the optimal behavior of both profit and nonprofit organizations.

For ease of exposition and brevity, we confine our discussion to the illustrative example that V&M use—a series of concerts of Indian music organized by a classical musical/dance association—but state our formulation in more general terms. The reader is referred to V&M's study for more information on the history and motivation of this problem.

MODEL OVERVIEW

Number of Events

In V&M's study, management contemplates presenting a series of ten different events. As is common with many service businesses, the cost of providing these events varies more directly with the capacity to provide service than with the number of people attending the events. In particular, V&M estimate cost per performance at $1,200 and cost per ticket sold (or event attended) at $0.

A critical element of profitability for managers of service businesses is capacity management. Although V&M gathered market research data on the number of different events (up to ten), that respondents would be likely to attend, this does not require the organization to offer ten separate events. Moreover, the pricing policy used—pure compo-

nents, pure bundling, or mixed bundling—influences the optimal number of events to schedule.

Nonprofit Organizations

Nonprofit organizations account for approximately 10% of the paid labor force (Weisbrod 1988, pp. 62–67), as well as volunteer time donated by approximately 50% of the adult U.S. population (Weisbrod 1988, p. 175). Nonprofit organizations are a significant and growing force in the economy and are particularly prevalent in the performing arts.

By definition, nonprofit organizations have an objective function different from that of profit-maximizing business-es. In a cross-sectional study based on records filed with the Internal Revenue Service, Steinberg (1986, p. 508) finds that "welfare, education, and arts firms are service maxi-mizers [but] that health firms are budget maximizers...." Jacobs and Wilder (1984) in a study of Red Cross blood ser-vice units find empirical support for an objective in which output is maximized subject to a break-even constraint. Weinberg (1980) postulates an objective function (within a defined product-market) of usage maximization subject to a nondeficit constraint. Because of this support, we adopt a modified version of Weinberg's (1980) objective function for the nonprofit organization, which enables the nonprofit organization to consider number of users, as well as total usage.

STATEMENT OF THE MODEL

Following V&M, we investigate three basic pricing strategies (i.e., single tickets, pure bundling, and mixed bundling strategies) for both the profit and nonprofit sectors. Details of these cases are presented with a definition of the notation in Appendix 2. Here, we present the pure bundling case for nonprofit organizations, because it illustrates the basic issues to be studied. We continue with the notation used by V&M.

Profit maximizing firm. An important component of V&M's model is the assumption, supported with empirical data, that members of the target market make independent probabilistic decisions about the number of events to attend and have a reservation price for each event. Let X and Y be random variables, where X is the number of performances a person can attend and Y is a person's mean reservation price across events. The proportion $P_r(p_b;n)$ of the market willing to buy a season ticket consisting of n events is given by,

$$(1) \qquad P_r(p_b;n) = \sum_{i=1}^{n} \text{Prob}(X = i) \text{Prob}\left(Y \geq \frac{p_b}{i}\right)$$

$$+ \text{Prob}(X > n) \text{Prob}\left(Y \geq \frac{p_b}{n}\right),$$

where p_b is the price of a bundle of n events.

The first term in Equation 1 recognizes the probability of wanting to attend i ($\leq$ n) events. The second term is added to recognize explicitly the probability that people willing to attend more than n events will still be in the market if their reservation price is not exceeded. Let $\Pi(p_b;n)$ denote profits to obtain the following:

$$(2) \qquad \Pi(p_b;n) = P_r(p_b;n) Mp_b - nE,$$

where E is the fixed cost per performance, which is assumed to be equal across all events as in V&M's study, and M is the total size of the market.

For any given value of n, the profit maximizing price, p_b^*, of a bundle of n events can be determined numerically. A grid search over all feasible values of n yields the optimal number, n*, of performances.

Nonprofit organization. In pure bundling, the seller offers only a package of n events. However, buyers of the bundle do not necessarily attend n events. The expected total atten-dance by buyers of an n event bundle, $A(p_b;n)$, is determined by,[1]

$$(3) \qquad A(p_b;n) = \left[\sum_{i=1}^{n} i \, M \, \text{Prob}(X = i) \text{Prob}\left(Y \geq \frac{p_b}{i}\right)\right]$$

$$+ n \, M \, \text{Prob}(X > n) \text{Prob}\left(Y \geq \frac{p_b}{n}\right).$$

For the nonprofit organization, according to Weinberg's (1980) study, the goal is to maximize attendance without incurring a deficit. Thus, the nonprofit's problem can be stated as,[2]

$$(4) \qquad \text{Maximize } A(p_b;n),$$

$$\text{Subject to } \Pi(p_b;n) = 0.$$

In Weinberg's (1980) study, the nonprofit's objective func-tion may be overly restrictive. For example, a nonprofit arts organization may be more interested in how many (differ-ent) people it reaches, rather than in how many total tickets are sold. Analogous to the issue of reach and frequency in media selection, arts organizations may value highly the first few exposures to their performances. To capture these effects parsimoniously, we propose the following model of weighted attendance:

$$(5) \qquad W(p_b;n,\gamma) = \left[\sum_{i=1}^{n} w_i(\gamma) \, M \, \text{Prob}(X = i) \text{Prob}\left(Y \geq \frac{p_b}{i}\right)\right]$$

$$+ w_n(\gamma) \, M \, \text{Prob}(X > n)\text{Prob}\left(Y \geq \frac{p_b}{n}\right),$$

where $w_i(\gamma) = \sum_{j=1}^{i} \gamma^{j-1}$, γ is a weighting constant such that $0 \leq \gamma \leq 1$, and n is the total number of events in the bundle.

To illustrate $w_i(\gamma)$, we consider a consumer who attends three performances. When $\gamma = 1$, the weight $w_i(\gamma) = 3$, and each performance attended is given equal weight. Thus, the weighted attendance given in Equation 5 is equal to the total attendance in Equation 3. When $\gamma \to 0$, $w_i(\gamma) \to 1$, and attendance by the same person beyond the first time is given zero weight. In this case, weighted attendance is equivalent to the total number of people who attend at least one perfor-mance. Varying γ between 0 and 1 varies the emphasis

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[1]For details of estimation issues see Appendix A.

[2]Nonprofit organizations also perform fund-raising activities and use D, the net donations received (D = total contributions minus fund-raising expenditures), to offset deficits incurred in operations. In such cases the value of D can be added to the left-hand side of equations 4 and 6, and the subsequent Lagrangian can be modified appropriately. The comparative results for the nonprofit and profit cases reported here will continue, but with the differences strengthened depending on the magnitude of D.

Table 1

PROFIT–MAXIMIZING FIRM

OPTIMAL PROFIT VERSUS NUMBER OF PERFORMANCES

	Optimal Profits $		
n	Single	Bundle	Mixed
5	13717	11831*	17773
6	13909*	11284	17777*
7	13425	10212	16945
8	12533	9012	15844
9	11443	7812	14657
10	10274	6612	13456

*Maximum profit.

between number of users (reach) and total usage (frequency).[3]

The nonprofit's objective function can now be stated as,

(6)   Maximize $W(p_b;n,\gamma)$,

Subject to $\Pi(p_b;n) = 0$.

If it is restated as a Lagrangian, we have $L = W + \lambda\Pi$.

Optimal solutions. As in V&M's study, a closed form solution for p_b cannot be stated. However, for any given value of n, an optimal price equation can be specified. For the profit-maximizing organization, we have,

(7) $\dfrac{d\Pi}{dp_b} = p_b \dfrac{dP_r(p_b;n)}{dp_b} + P_r(p_b;n) = 0,$

whereas for the nonprofit, we obtain,

(8) $\dfrac{\partial L}{\partial p_b} = \dfrac{\partial W}{\partial p_b} - \lambda\left[p_b \dfrac{\partial P_r(p_b;n)}{\partial p_b} + P_r(p_b;n) \right] = 0,$

$\dfrac{\partial L}{\partial \lambda} = P_r(p_b;n) \, M \, p_b - n \, E = 0.$

For the nonprofit organization, fixed costs (i.e., the cost to offer n events) affect optimal prices. Unlike the profit-maximizing case, in which fixed costs only determine whether to enter the business, the zero-profit constraint implies that prices are set to (just) cover all costs, including fixed costs.

[3]Although we confine our study to values of γ between 0 and 1, a value of $\gamma > 1$ implies that exposure to the second and other subsequent performances are more important to the organization than the first. Values of $\gamma > 1$ might occur if the organization is attempting to build a subscriber base or deepen audience members' appreciation of the arts.

RESULTS AND DISCUSSION

As we expected, the flexibility to choose the optimal number of products to offer enables the organization studied by V&M to have a substantial increase in profits. Moreover, nonprofit managers make different decisions from profit maximizers. Detailed results are presented in Tables 1 through 4, and we discuss them subsequently.

Number of Events: Profit Maximizing Firm

The optimal profits that a profit-maximizing firm could obtain by staging different numbers of events for each of the three pricing strategies are shown in Table 1. The optimal number of events is six for the single ticket strategy (maximum profit = $13,909), five for pure bundling (maximum profit = $11,831), and six when mixed bundling is followed (maximum profit = $17,777). As in V&M's study, mixed bundling remains the optimal policy overall.

The impact on profits of reducing the number of events from the initially planned value of ten is substantial; as is shown in Table 1, at the optimum, a profit improvement of more than 25% is obtained for the mixed bundling case. This is primarily because relatively few people are likely to attend more than six events, so most of the profit improvement comes from cost savings.

In Table 2, part A are the profit-maximizing firm's optimal prices and the resulting attendance for each pricing strategy for the following scenarios: n = n* (optimal number of events held); n = 8 (a reference case); and n = 10, (V&M's case). A comparison of optimal prices across the three policies reveals, as in V&M's study, that for any n, the single ticket and bundle prices for mixed bundling always exceed the corresponding prices in the other two strategies. This is due to the increased discrimination that is possible through mixed bundling. Note also that the optimal prices for both the single ticket and pure bundling cases do not vary when eight or more events are scheduled and the impact on attendance is less than 1%. This is due to the skewed nature of the attendance distribution.

Surplus maximization. To examine the pricing and bundling problem from a perspective other than that of the profit-maximizing firm, we examine the total surplus, which consists of the firm's surplus or profit and the consumer surplus.[4] Consumer surplus accounts for the gain that users obtain from paying a price below their reservation price. For pure components, the optimal number of events to schedule

[4]Mathematical details are presented in Appendix B.

Table 2

RESULTS FOR PROFIT MAXIMIZING AND NONPROFIT FIRMS: PRICE AND ATTENDANCE FOR DIFFERENT POLICIES

	Price $				Attendance		
	Single	Bundle	Mixed		Single	Bundle	Mixed
Case			Single	Bundle			
A. Profit n = n*	10.48	35.50	12.56	50.24	2015	2231	2407
Profit n = 8	10.48	41.21	12.20	60.99	2112	2403	2361
Profit n = 10	10.48	41.21	12.20	60.99	2126	2423	2386
B. Nonprofit n = n* = 8	3.05	13.50	4.85	14.01	3145	3182	3202

is seven, and the optimal single ticket price is zero dollars. What is the reasoning behind the zero price? Lowering the price to zero increases attendance and contributes to increased consumer surplus. In addition to the new consumers, existing consumers also experience a higher surplus (because $r_s - p_s$ increases). The increase in consumer surplus offsets the decrease in firm profit, which provides an incentive to lower p_s to zero. Below this level, however, surplus is merely transferred from the firm to the consumer, which keeps total surplus constant.

For pure bundling, the optimal number of events to schedule is seven, but the optimal price is indeterminate between zero and six dollars. This indeterminate region is due to the particular reservation price probability distribution for bundles, in which no consumer demand is lost for bundles priced between zero and six dollars. In this region, total surplus is constant, but is transferred between consumers and the firm. By extension, the optimal price for mixed bundling is zero dollars.

Additional options—part bundles. As is often seen in practice, and as V&M (p. 506) suggest, arts organizations often offer bundles consisting of $n_k < n$ events. Although a variety of n_k strategies can be pursued, for illustrative purposes, we confine ourselves to the $n_k = n/2$ mixed bundling case. We name this case *amplified mixed bundling.*[5] As expected, amplified mixed bundling outperforms mixed bundling for V&M's data. The case n = 6 is still optimal, with profits increasing to $17,974, or by 1%. (Note that there was a greater comparative profit increase for the nonoptimal cases; for example, when n = 8, profits increased to $17,238, or by 10%.) Consistent with the comparative results for mixed bundling versus pure bundling or pure components, the optimal single-ticket price ($12.01) and n-event bundle price ($60) for amplified mixed bundling is higher than the comparable prices for mixed bundling. The optimal price for the n_k (three) event bundle is $36. Similar results hold for other values of n.

Nonprofit Organizations

Nonprofit organizations seeking to maximize usage while "breaking even" pursue different strategies than similarly situated profit-maximizing firms. We first discuss the results for the nonprofit that maximizes total attendance, then examine the results for weighted attendance-maximization, and conclude by considering the impact on these results of using surplus maximization as an objective function.

Attendance maximization. For all three pricing policies, the optimal number of events to schedule is eight, which represents more performances than a profit-maximizing firm would hold. Optimal prices and corresponding attendance figures are shown in Table 2, part B.

Maximum attendance does not occur with the maximum number of performances scheduled. As is shown in Table 3, attendance for each pricing strategy declines when more than eight events are scheduled. Because the nonprofit organization must cover its costs, the effect on prices of increasing the number of events is large enough to result in a downturn in expected attendance. This is a critical difference from the profit-maximizing case, in which prices do not

[5]Mathematical details are presented in Appendix C.

Table 3
NONPROFIT ORGANIZATION: ATTENDANCE VERSUS NUMBER OF PERFORMANCES

n	Expected Attendance		
	Single	Bundle	Mixed
5	2838	2858	2862
6	3029	3053	3063
7	3119	3150	3158
8	3145*	3182*	3202*
9	3137	3178	3200
10	3113	3162	3196

*Maximum attendance.

change with capacity costs as long as the firm has sufficient profits to cover them. Optimal prices for the nonprofit case change with the number of events scheduled. This is in contrast to the single ticket case for a profit-maximizing business, wherein the optimal price is $10.48 for any n.

For V&M's data, the optimal price charged by a nonprofit is approximately 70% below that charged by a profit-maximizing business—a substantial difference. Although the effects are not always so dramatic, the nonprofit always charges a lower price (Weinberg 1980).[6]

Weighted-attendance maximization. When the nonprofit seeks to maximize weighted attendance subject to a nondeficit constraint, the broad pattern of results for the unweighted attendance case holds, but in some cases, fewer numbers of events are scheduled. The results for the pure bundling case for $\gamma = 0$ to 1 are shown in Table 4.

At the extreme, when only reach is important ($\gamma = 0$), the optimal schedule consists of only one event for V&M's data.[7] As γ increases from 0 to 1, frequency of exposure gains importance, and the optimal number of events to schedule increases as well (see Table 4), reaching the unweighted optimal of eight events when $\gamma = .9$. The incremental cost of holding additional events also leads to higher prices, as is shown in Table 4, though the effective ticket price (optimal bundle price divided by optimal number of events) remains relatively stable. Because of the nondeficit constraint, once the optimal number of events is determined, the price is set at the lowest possible value and does not vary with γ.

For the nonprofit organization studied, pure bundling dominates the single ticket strategy in terms of expected attendance (see Table 3), unlike in the profit-maximizing firm (Table 1) for which the single ticket strategy is better. However, as in the profit-maximizing case, mixed bundling is optimal even for the nonprofit firm. Our calculations also

[6]The results derived here and in V&M's study are for monopoly providers of services. In the case of perfect competition, more similarities might appear, because the profits of businesses would be driven to zero. However, the motivation for entering and remaining in the market would be different, as would the treatment of fixed costs.

[7]This result assumes that (1) the organization does not face any capacity constraints and (2) all performances are equivalent and interchangeable (i.e., they are equally preferred and share the same reservation price distribution). As in V&M's model, this assumes that consumers' probability distribution for number of events is not related to the specific time at which events are held. Otherwise, an organization might maximize reach by holding more than one event.

Table 4

NONPROFIT ORGANIZATION: MAXIMIZATION OF WEIGHTED ATTENDANCE: PURE BUNDLING CASE

Weighting Parameter(γ)	Optimal Number of Events	Optimal Bundle Price ($)	Effective Ticket Price ($)*	Total Attendance	Weighted Attendance
0.0	1	1.63	1.63	738	738
.1	3	4.85	1.61	2050	818
.2	4	6.48	1.62	2530	909
.3	4	6.48	1.62	2530	1018
.4	5	8.17	1.63	2858	1151
.5	5	8.17	1.63	2858	1317
.6	6	9.90	1.65	3053	1529
.7	7	11.66	1.67	3150	1799
.8	7	11.66	1.67	3150	2146
.9	8	13.50	1.69	3182	2597
1.0	8	13.50	1.69	3182	3182

*Effective ticket price = Optimal bundle price/Optimal number of events.

show that similar results are obtained with the weighted-attendance maximization criteria for a wide range of γ values.

Surplus maximization. An alternative objective function for a nonprofit organization would be maximizing (total) surplus subject to a nondeficit constraint. This is equivalent to maximizing consumer surplus subject to a nondeficit constraint. In some ways, maximizing consumer surplus may be a troubling objective for some nonprofit organizations, because it places a premium on consumers willing to pay the highest price. This is an unlikely focus for many nonprofits, because higher-income people are generally, *ceteris paribus*, more willing to pay higher prices. Nevertheless, for the data studied here, both pure bundling and pure components lead to an optimal schedule of seven events, with $p_b = \$11.66$ and $p_s = \$2.69$. This is the same number of events as in the unconstrained surplus maximization case, but total surplus (and consumer) surplus are lower because of the nondeficit constraint.

CONCLUSION

Our results demonstrate that nonprofit organizations tend to make substantially different marketing mix decisions than similarly situated profit-maximizing firms. For the organization studied here, an objective of usage maximization subject to a nondeficit constraint leads to lower prices and a greater number of events than a profit-maximizing firm would offer. However, the nonprofit offers neither the greatest possible number of events nor the lowest possible price, because neither strategy completely matches the objectives (weighted-usage maximization) and nondeficit constraints of the organization. Moreover, unlike the situation in a for-profit business, the optimal price depends on the fixed costs, because the nondeficit constraint includes total costs. Although our research extends V&M's work in several directions, the limitations they state (pp. 505–506) are still broadly applicable here. Considering the effects of a multi-period time horizon (to allow time to build an audience), uncertainty, donations, and competition among nonprofits[8]

[8]For a discussion of managerial issues, see Gallagher and Weinberg (1991), and for a more theoretical approach, see Kim and Weinberg (1991).

would all be interesting, but the directionality of the comparative results is unlikely to change substantially.

We develop a compact objective function that enables nonprofits to place emphasis on *number of users* versus *total usage*. As is illustrated here, the relative weight placed on these factors can result in different decisions with regard to price and the number of events to schedule. The emphasis to place on number of users versus usage is an empirical question to be resolved for each organization. In an applied setting, marketing research techniques can assess the relative emphasis that stakeholders place on these two factors.

From a societal view, organizations can be examined in terms of the total surplus they produce. We determine the optimal policies for maximizing total surplus in for-profit businesses and nonprofit (deficit-constrained) organizations. Because a deficit-constrained nonprofit organization has less freedom than an organization that can maximize the combined total of consumer and firm surplus, the former generates a lower total surplus (along with higher prices and fewer events) than an unconstrained business would. This analysis leaves unanswered the question of how the firm's deficit should be funded. Society's expectations of organizations and the rationale for their existence are appealing research issues (see Weisbrod 1988). As is apparent and has been illustrated here, marketing mix decisions are sensitively dependent on the organization's objective function.

We extend V&M's model to incorporate the decision of what number of events to schedule. Operationally, this analysis requires a minimal increase in mathematical complexity and no more effort than the original model requires. Moreover the number of events to schedule has a substantial impact on profitability (and attendance) and, thus, is worthy of management attention.

The optimality of mixed bundling, and in turn, amplified mixed bundling might have been anticipated by appealing to the literature on product-line marketing and price discrimination (Moorthy 1984; Shaked and Sutton 1982). A monopolist is able to extract more profits and serve the market better with two products than it would with one; mixed bundling is analogous to the two-product case, whereas pure bundling or pure components is similar to a one-product strategy. The advantage of mixed bundling appears to hold for nonprofit organizations, as well. However, these analy-

ses omit the operating (or transaction) costs of administering a mixed-bundling system. It is surely more difficult to sell and service single *and* subscription tickets than provide only one kind of ticket. If these costs are significant enough, then (amplified) mixed bundling may not be optimal. In the case of the nonprofit organization studied here, mixed bundling provides an increase of less than 2% in total attendance; thus, the value of this option appears to be limited. The utilization of mixed bundling on both theoretical grounds and in managerial settings remains an interesting research question.

APPENDIX A

Venkatesh and Mahajan (1993, Question 8, p. 507) asked respondents how many of ten events they expected to attend. An answer of six might reflect taste (I like Indian music, but I only want to go to six Indian music concerts a year) and time constraints or uncertainty (given my busy schedule, I only expect to have six Saturdays free to attend Indian music). In addition, from a list of ten specific concerts, the respondents also indicated (Question 9, p. 507) which concerts they would attend (without specifying the dates). The number of events to be attended was totaled, and the minimum of the answers to Question 8 and Question 9 was then used to estimate the market-level Weibull function. Venkatesh and Mahajan essentially assumed that any limitation from Question 8 first affects the events not chosen and then is applied to the desired events. However, V&M could directly use their data to determine the expected attendance and then proceed sequentially. Lacking their data, we assume that persons who said they would attend $k \geq n$ events would attend n events if n were offered. This is similar in spirit to the approach that V&M use. (In effect, we are assuming that the answer to Question 9 is driving the attendance frequency issue and that performance type rather than specific performer is important, at least at the aggregate level. For evidence on the latter point in the context of different types of performing arts events, see Weinberg [1986].) Moreover, if the original purpose had included the goal of determining the number of events to be presented, then different questions would have been asked. We recognize that our approach is only an approximation. Because our purpose is to illustrate that number of events to be presented is a meaningful issue, not the particular numerical result, we believe that this approximation is reasonable.

APPENDIX B

Let M be the market potential at 750; p_s be the price for a single ticket; p_b be the price for a bundle (season ticket); n be the number of performances in the season; r_s be the performance-wise reservation price; r_m be the mean reservation price; F be the distribution function for the likely number of performances attended; G and g be the distribution function and density function, respectively, for the performance-wise reservation price; and H and h be the distribution function and density, respectively, for the mean reservation price. The objective functions to be maximized can be represented as follows:

Single Tickets

Profit Maximization. The expected attendance for n performances is,

$$A(p_s;n) = \left\{ \sum_{i=1}^{n} iM[F(i + 1) - F(i)][1 - G(p_s)] \right\}$$
$$+ nM[1 - F(n + 1)][1 - G(p_s)],$$

and the expected profit is given by $\Pi(p_s;n) = A(p_s;n)p_s - 1200n$.

Surplus Maximization. The consumer surplus for n performances is,

$$C(p_s;n) = \left\{ \sum_{i=1}^{n} iM[F(i + 1) - F(i)] \int_{p_s}^{\infty} (r_s - p_s)g(r_s)dr_s \right\}$$
$$+ nM[1 - F(n + 1)] \int_{p_s}^{\infty} (r_s - p_s)g(r_s)dr_s,$$

and the total surplus is given by $T(p_s;n) = \Pi(p_s;n) + C(p_s;n)$.

Nonprofit—Weighted-Attendance Maximization. The expected weighted attendance for n performances is given by

$$W(p_s;\gamma,n) = \left\{ \sum_{i=1}^{n} w_i(\gamma)M[F(i + 1) - F(i)][1 - G(p_s)] \right\}$$
$$+ w_n(\gamma)M[1 - F(n + 1)][1 - G(p_s)],$$

where, $w_i(\gamma) = \sum_{j=1}^{i} \gamma^{j-1}$. The optimal price and weighted attendance for a given n can be obtained by maximizing the Lagrangian, $L(p_s,\lambda;n,\gamma) = W(p_s;\gamma,n) + \lambda\Pi(p_s;n)$.

Nonprofit—Surplus Maximization. The Lagrangian is given by $L(p_s,\lambda;n) = C(p_s;n) + \lambda\Pi(p_s;n)$.

Pure Bundling

Profit Maximization. The proportion of the market willing to purchase the season ticket for n performances is given by

$$P_r(p_b;n) = \left\{ \sum_{i=1}^{n} [F(i + 1) - F(i)][1 - H\left(\frac{p_b}{i}\right)] \right\}$$
$$+ [1 - F(n + 1)]\left[1 - H\left(\frac{p_b}{n}\right)\right],$$

and the corresponding profit $\Pi(p_b;n) = MP_r(p_b;n)p_b - 1200n$.

Surplus Maximization. Consumer Surplus is given by

$$C(p_b;n) = \left\{ \sum_{i=1}^{n} iM[F(i + 1) - F(i)] \int_{l(i)}^{\infty} \left(r_m - \frac{p_b}{i}\right)h(r_m)dr_m \right\}$$
$$+ nM[1 - F(n + 1)] \int_{l(n)}^{\infty} \left(r_m - \frac{p_b}{n}\right)h(r_m)dr_m,$$

where, the lower limit l(i) of the integral is given by

$$l(i) = \begin{cases} \dfrac{p_b}{i} & \text{if } \dfrac{p_b}{i} > 6, \\ 6 & \text{otherwise.} \end{cases}$$

The total surplus is obtained as $T(p_b;n) = \Pi(p_b;n) + C(p_b;n)$.

Nonprofit—Weighted Attendance Maximization. The expected weighted attendance is given by

$$W(p_b;\gamma,n) = \left\{ \sum_{i=1}^{n} w_i(\gamma)M[F(i + 1) - F(i)]\left[1 - H\left(\frac{p_b}{n}\right)\right]\right\}$$
$$+ w_n(\gamma)M[1 - F(n + 1)]\left[1 - H\left(\frac{p_b}{n}\right)\right],$$

which yields the Lagrangian, $L(p_b,\lambda;\gamma,n) = W(p_b;\gamma,n) + \lambda\Pi(p_b;n)$.

Nonprofit—Surplus Maximization. The Lagrangian is given by $L(p_b,\lambda;n) = C(p_b;n) + \lambda \Pi(p_b;n)$.

Mixed Bundling

Profit Maximization. Following V&M, define m(i) as

$$\begin{cases} 1 \text{ if } \frac{p_b}{i} < p_s, \\ \\ 0 \text{ otherwise.} \end{cases}$$

Then, given n performances in a season, the expected number of bundles sold is given by

$$B(p_b,p_s;n) = \left\{ \sum_{i=1}^{n} m(i)M[F(i + 1) - F(i)]\left[1 - H\left(\frac{p_b}{i}\right)\right]\right\}$$
$$+ m(n)M[1 - F(n + 1)]\left[1 - H\left(\frac{p_b}{n}\right)\right].$$

To determine the expected number of single tickets, define $J = M[F(i + 1) - F(i)][1 - G(p_s)]$ and $K = M[1 - F(n + 1)][1 - G(p_s)]$. The number of single tickets is given by

$$S(p_s,p_b;n) = \left\{ \sum_{i=1}^{n} im(i)JH\left(\frac{p_b}{i}\right) + i[1 - m(i)]J\right\}$$
$$+ nm(n)KH\left(\frac{p_b}{n}\right) + n[1 - m(n)]K,$$

whereas the profit $\Pi(p_b,p_s; n) = S(p_s,p_b;n)p_s + B(p_b,p_s;n)p_b - 1200n$.

Surplus Maximization. The consumer surplus accruing to those consumers who buy single tickets is given by

$$CS(p_s,p_b;n) = \int_{p_s}^{\infty}(r_s - p_s)g(r_s)dr_s \left(\sum_{i=1}^{n} i[F(i + 1) - F(i)]M\right.$$
$$\left\{m(i)H\left(\frac{p_b}{i}\right) + [1 - m(i)]\right\} + n[1 - F(n + 1)]M$$
$$\left.\left\{m(n)H\left(\frac{p_b}{n}\right) + [1 - m(n)]\right\}\right).$$

The consumer surplus accruing to consumers who choose bundles is given by

$$CB(p_b,p_s;n) = \left\{ \sum_{i=1}^{n} iM[F(i + 1) - F(i)]m(i)\int_{I(i)}^{\infty}\left(r_m - \frac{p_b}{i}\right)h(r_m)dr_m\right\}$$
$$+ nM[1 - F(n + 1)]m(n)\int_{I(n)}^{\infty}\left(r_m - \frac{p_b}{n}\right)h(r_m)dr_m,$$

where l(i) is as previously defined. Hence, consumer surplus is $C(p_s,p_b;n) = CS(p_s,p_b;n) + CB(p_b,p_s;n)$, and total surplus is the sum of consumer surplus and profits.

Nonprofit—Weighted Attendance Maximization. The expected weighted attendance from consumers who buy season tickets is given by

$$WB(p_b,p_s;\gamma,n) = \left\{ \sum_{i=1}^{n} w_i(\gamma)m(i)M[F(i + 1) - F(i)]\left[1 - H\left(\frac{p_b}{i}\right)\right]\right\}$$
$$+ w_n(\gamma)m(n)M[1 - F(n + 1)]\left[1 - H\left(\frac{p_b}{n}\right)\right],$$

whereas the expected weighted attendance from consumers who buy single tickets is given by

$$WS(p_s,p_b;\gamma,n) = \left\{ \sum_{i=1}^{n} w_i(\gamma)m(i)JH\frac{p_b}{i} + w_i(\gamma)[1 - m(i)]J\right\}$$
$$+ w_n(\gamma)m(n)KH\frac{p_b}{n} + w_n(\gamma)[1 - m(n)]K.$$

Then, the Lagrangian is given by $L(p_b,p_s,\lambda;n,\gamma) = WS(p_b,p_s;\gamma,n) + WB(p_b,p_s;\gamma,n) + \lambda\Pi p_b,p_s;n)$.

Nonprofit—Surplus Maximization. The Lagrangian is given by $L(p_b,p_s,\lambda;n) = C(p_b,p_s;n) + \lambda\Pi p_b,p_s;n)$.

APPENDIX C

Amplified Mixed Bundling

In this strategy, the arts organization simultaneously offers (1) a bundle of n tickets, (2) a bundle of n/2 tickets, and (3) single tickets. Assume that n is even. Let p_h be the price for half bundles; p_b be the price for the full bundle of n tickets, and p_s be the price for a single ticket. Also, let i be the number of events a customer has time to attend. We first start with the relatively simple situation of a customer with $i \le n/2$.

Situation I: (i ≤ n/2)

Because we expect p_h/i to be less than p_b/i, consumers with $i \le n/2$ essentially choose between buying i single tickets or buying the half bundle and make this case similar to simple mixed bundling. Let sh(i) = 1 if $p_h/i < p_s$; 0 otherwise. The expected number of half bundles bought by consumers with $i \le n/2$ is given by

$$HB11 = \sum_{i=1}^{n/2} sh(i)M[F(i + 1) - F(i)]\left[1 - H\left(\frac{p_h}{i}\right)\right],$$

and the number of single tickets is given by

$$S11 = \sum_{i=1}^{n/2} iM[F(i + 1) - F(i)][1 - G(p_s)]\left\{sh(i)H\left(\frac{p_h}{i}\right) + [1 - sh(i)]\right\}.$$

Next, we consider the situation facing consumers with $i > n/2$.

Situation II: (i > n/2)

Consumers with i > n/2 choose between (1) i single tickets, (2) half bundle + (i − n/2) single tickets, and (3)a full bundle. Consumers who buy a combination of the half bundle and single tickets pay an effective price per performance of ep(i) = $[p_h + (i − n/2)p_s]/i$. The relative ordering of the prices, p_b/i, ep, and p_s yield three distinct cases.

Case 1. $p_b/i < p_s$ and $p_b/i < ep(i)$.

Let bs(i) = 1 if $p_b/i < p_s$; 0 otherwise, and bh(i) = 1 if $p_b/i < ep(i)$; 0 otherwise. In this case, the expected number of full bundles bought is given by

$$B21 = \sum_{i = n/2 + 1}^{n} bs(i)bh(i)M[F(i + 1) - F(i)]\left[1 - H\left(\frac{P_b}{i}\right)\right]$$

$$+ \left\{bs(n)bh(n)M[1 - F(n + 1)]\left[1 - H\left(\frac{P_b}{n}\right)\right]\right\}.$$

If the mean reservation price $p_m < p_b/i$, then the expected number of single tickets bought is given by

$$S21 = \sum_{i = n/2 + 1}^{n} ibs(i)bh(i)M[F(i + 1) - F(i)]H\left(\frac{P_b}{i}\right)G(p_s)$$

$$+ nbs(n)bh(n) M [1 - F(n + 1)]H\left(\frac{P_b}{n}\right)G(p_s).$$

Case 2. $ep(i) < p_b/i$ and $ep(i) < p_s$.

Let $hss(i) = 1$ if $ep(i) < p_s$; 0 otherwise. The expected number of half bundles bought is given by

$$HB22 = \sum_{i = n/2 + 1}^{n} [1 - bh(i)]hss(i)M[F(i + 1) - F(i)]\{1 - H[ep(i)]\}$$

$$+ [1 - bh(n)]hss(n)M[1 - F(n + 1)]\{1 - H[ep(n)]\},$$

and the expected number of single tickets is given by

$$S22 = \sum_{i = n/2 + 1}^{n} i[1 - bh(i)]hss(i)M[F(i + 1) - F(i)]H[ep(i)]G(p_s)$$

$$+ n[1 - bh(n)]hss(n)M[1 - F(n + 1)]H[ep(n)]G(p_s).$$

Case 3. $p_s < p_b/i$ and $p_s < ep(i)$.

In this case, only single tickets are chosen and the expected number of single tickets is given by

$$S23 = \sum_{i = n/2 + 1}^{n} i[1 - bs(i)][1 - hss(i)]M[F(i + 1) - F(i)]G(p_s)$$

$$+ n[1 - bs(n)][1 - hss(n)]M[1 - F(n + 1)]G(p_s).$$

Thus, the expected number of tickets of each type sold is given by (1) single tickets: $S(n) = S11 + S21 + S22$; (2) half bundles: $HB(n) = HB11 + HB22$, and (3) full bundles: $B(n) = B21$. The appropriate objective functions are constructed to calculate the optimal prices for each type of ticket.

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