JAMES R. BETTMAN, NOEL CAPON, and RICHARD J. LUTZ*

Research on multi-attribute attitude models has relied on correlational methods. An analysis of variance paradigm applied to individual level data supports the multiplicative assumption of multi-attribute models. A single-attribute form of the Fishbein model is found to be superior to a similar version of the "adequacy-importance" model.

Cognitive Algebra in Multi-Attribute Attitude Models

INTRODUCTION

During the past several years, one of the more intensively studied areas in marketing and consumer research has been multi-attribute models of attitude [36]. The basic proposition of these models is that consumers form attitudes toward products on the basis of product attributes, a formulation which has considerable implications for marketing strategies. While a variety of alternative specifications of the multi-attribute model have been employed, the most prominent have been the so-called "adequacy-importance" model [10, 11, 14, 17, 31] and the approach advocated by Fishbein [15, 18, 22, 35]. Empirical research directed at validating these various forms of the multi-attribute model has commonly been based on cross-sectional correlation designs, although more recently individual level analysis has been strongly advocated [11, 12]. This article presents an alternative approach for investigating the adequacy-importance and Fishbein models at the individual level.

The Fishbein Model may be formulated as:

$$A_j = \Sigma_{i=1}^{n} a_i b_{ij}, \tag{1}$$

where:

A_j = attitude toward brand j,
a_i = evaluative aspect of attribute i, its goodness or badness,
b_{ij} = strength of belief that brand j possesses attribute i,
n = number of attributes.

In a similar manner, the adequacy-importance model may be expressed as:

$$A_j = \Sigma_{i=1}^{n} I_i B_{ij}, \tag{2}$$

where:

A_j = attitude toward brand j,
I_i = importance weight given attribute i,
B_{ij} = belief as to the extent to which attribute i is offered by brand j,
n = number of attributes.

Controversy has developed over which of these models is more valid for use in marketing, and attempts have been made to compare the two approaches. Bass [8], Sheth [30], Talarzyk [32], and Cohen, Fishbein, and Ahtola [17] compared the models on theoretical and practical grounds, but no empirical work was reported in these articles. Mazis and Klippel [25] compared the two models on the basis of cross-sectional analysis, using the coefficient of determination (r^2) as the criterion. They found that the adequacy-importance model was slightly, but consistently, better in terms of r^2. However, as Bass and Wilkie [11] have pointed out, cross-sectional analyses are subject to many pitfalls, since individual differences can distort results. In addition, Birnbaum [13] has argued forcefully that judging models via correlational procedures can lead to incorrect inferences, and he provides examples in which data sets correlate better with predictions from an incorrect model than with predictions from the "true" model which was used to generate the data. Based on these considerations, the

*James R. Bettman is Associate Professor and Noel Capon and Richard J. Lutz are Assistant Professors at the Graduate School of Management, University of California, Los Angeles. They gratefully acknowledge support of the Center for Marketing Studies at the Graduate School of Management. The ordering of the authors' names is alphabetical; all three contributed equally to the project.

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thesis of this research is that a rigorous comparison of the two models has not yet been reported. The purpose of this study is to provide such a comparison. Note that the Rosenberg [26] model is not tested, as it has not been widely used in marketing in its original form.

In comparing the models, the viewpoint taken here is that the most realistic and theoretically consistent approach is to analyze individual level data. Individual level analysis does have limitations, of course. The amount of data required from each individual may be large, and since individuals are not necessarily stable in their processes, there may be considerable heterogeneity even within an individual.

The crucial elements of the models are believed to be the *components* they employ and the *composition rules* they assume, i.e., the way each model assumes that individuals combine the various pieces of information measured. The important aspects of the models' composition rules, as expressed by the basic algebraic assumptions of the models, are as follows:

1. The contribution to affect for each attribute is *multiplicative* for each model; e.g., $I_i B_{ij}$ and $a_i b_{ij}$.
2. The *powers* of the terms in the product for each attribute are assumed to be 1 for both models (although beliefs-only models have been suggested, where the power of a_i or I_i is assumed to be zero).
3. The product terms, $a_i b_{ij}$ and $I_i B_{ij}$, *add* over attributes to form an overall affect measure in both models.
4. The components themselves, a_i, b_{ij}, I_i, and B_{ij}, are *coded* in particular ways as part of the measurement process. Researchers using the Fishbein model have typically treated a_i and b_{ij} as bipolar (e.g., -2 to $+2$) scales, while adequacy-importance model researchers have tended to treat the scales as nonnegative (e.g., 1 to 5). Of course, differences between the two models in terms of the coding schemes used to assign numerical values to scale responses are not inherent to the specific mathematical formulations shown in (1) and (2) above. Obviously, a researcher using data gathered under the Fishbein format could elect to code the scales from 1 to 5, rather than -2 to $+2$, and likewise for the adequacy-importance model. In practice, however, much research using the Fishbein model [22, 35] has treated the scales as bipolar, while adequacy-importance model research has coded the scales on a unipolar basis.

These four aspects of composition rules may be referred to as *model algebra*. An examination of the algebraic assumptions indicates that the major difference in *model algebra* between the two formulations is with respect to point 4 above. This is more than a simple coding decision, however, for important psychological implications are involved. For example, Fishbein researchers assume that if a brand is believed *unlikely* to possess (say $b_{ij} = -2$) an attribute which is *very bad* ($a_i = -2$), then the contribution to affect is high, +4 in this case. Adequacy-importance model researchers have assumed that if a brand is believed *not to possess* ($B_{ij} = 1$, say) an attribute which is *unimportant* ($I_i = 1$), then the contribution to affect is low, +1. Thus, in summary, the two major distinctions between the models are: (1) the components used; and (2) the coding rules employed by researchers using the models.

Given these notions, a fundamental approach to comparison of the two models is to examine *directly* how consumers appear to make decisions about (i.e., form attitudes toward) products, based on perceptions of product attributes (b_{ij} or B_{ij}) and some sort of importance (I_i) or evaluation (a_i) associated with each of these attributes. Given a set of consumer decisions made with respect to a number of products, models (1) and (2) can be used to attempt to represent the process used in these decisions. To the extent that one model provides a consistently better representation than the other, then there is evidence that the *components* and *model algebra* of that model provide a closer approximation to the consumer's *cognitive algebra*, i.e., the composition rule the consumer appears to use [3]. The viewpoint of this study is that a basic approach to validation of the two competing models is to *examine how homogeneously consumers respond to the components used* for each model and to *examine how well model algebra for each of the two models corresponds to consumers' apparent cognitive algebra*, given the components used by each model. Regardless of what each model assumes consumers are doing, the consumers may very well be doing something different.

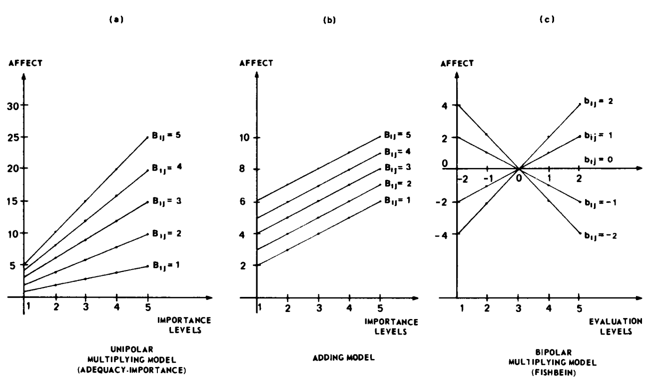
Note that the cognitive algebra used by subjects is *independent* of any assumptions about coding decisions. Cognitive algebra is simply the way subjects handle and combine the pieces of information given to them, and is an empirical phenomenon. Cognitive algebra *is* crucially affected by the specific components used in the models, however. The purpose of examining the coding assumptions is to aid in the interpretation of the empirical results. The main point of this article is to study how subjects actually combine data, and then to see how their behavior meshes with model assumptions. While the major factor which affects how subjects combine the data will likely be differences in the components used in the two models (i.e., importance versus evaluation, degree of belief versus likelihood), in the interpretation of the data relative to model algebra it becomes necessary to examine coding distinctions.

How would one examine consumers' cognitive algebra? As a first step, attention might be focused on the multiplication issue; i.e., to what extent is the multiplicative form of the multi-attribute model representative of consumers' personal combination rules? Using the method developed by Anderson [2, 3, 4, 5], investigation of the multiplication issue could be undertaken through an experiment in which subjects are asked to handle information for a *single* attribute. Suppose, for the adequacy-importance model, five distinct levels were defined for each of the I_i and B_{ij} components. By combining the 5 levels on each component, 25 distinct pairs of attribute information could be formed. A subject could then *rate* his affective reaction to (i.e., attitude toward) each pair of I_i and B_{ij} values. That is, subjects could be presented with profiles of the form:[1]

> You believe that Brand X is
> very high:__:__:X:__:__: very low
> in possessing a quality which you personally feel is:
> very important:__:__:__:__:X: not important at all.
> To you, using Brand X would be
> very favorable:__:__:__:__:__:__:__:__:__:__:__: very unfavorable.

Note that subjects would be *given* the levels of I_i and B_{ij} and asked to provide an affect rating. Alternatively, the product information could be presented on scales of the form used in operationalizing the Fishbein model. If the components and model algebra provide a valid representation of how subjects actually handle product information, then a definite pattern of affect ratings should emerge from the subjects' responses. However, the particular nature of this pattern would be expected to differ, depending upon how the subjects combine the information. Figure 1 illustrates the patterns of affect ratings which would

Figure 1
THEORETICAL COMBINATION RULES



be expected if the theoretical models were accurately representing subjects' information handling behavior.

The adequacy-importance model, since it assumes multiplication of I_i and B_{ij}, and typically assumes 1 to 5 (unipolar) coding for each component, implies a pattern similar to the form of Figure 1(a) if the subject's cognitive algebra is equivalent to the model algebra. For a fixed level of B_{ij}, the 5 I_i levels define a straight line with slope B_{ij}, and the overall pattern of the 25 data points is a diverging fan of straight lines in the upper right quadrant. The Fishbein model assumes multiplication also, but the typical bipolar coding of -2 to $+2$ for both a_i and b_{ij} implies a quite different pattern. For a fixed level of b_{ij}, a straight line with slope b_{ij} is obtained, and a pattern of the form of Figure 1(c) is found, a crossing pattern of straight lines. These two patterns are vastly different, as the ordinal properties of the data implied by each differ. Although (1) and (2)(together with the related coding decisions) imply patterns of this form, subjects' cognitive algebra may not be adequately represented by either model. For instance, one plausible alternative to multiplying the two components is to add them [4]. If subjects were *adding* I_i and B_{ij} (or, alternatively, a_i and b_{ij}), a pattern of the form shown in Figure 1(b), a set of parallel lines, would be obtained. The numerical values assigned to the components in these graphs correspond to the coding schemes typically used in attitude research. However, in the present analysis, the values are assigned for illustrative purposes only, because it is not necessary to make any coding assumptions in order to analyze the affect rating data. This is an important point and will be discussed further.

In summary, the present research examines *how subjects actually combine components given to them in developing a rating of affect.* Given these results, one can then examine the implications for the validity of the attitude models considered. First of all, the validity of the multiplicative assumption in the Fishbein and adequacy-importance models of attitude is considered. The design discussed earlier allows determination of whether subjects appear to be multiplying the two components, as detailed in the following sections. Second, it is important to examine the *patterns* of results obtained for each of the two models. If there is relatively greater heterogeneity of response patterns for one model than for the other, then the components used in that model are presumably more ambiguous. Finally, a third issue, although of lesser importance than the other two issues, is whether the specific coding rules used by researchers appear to correspond to the implicit coding (i.e., unipolar or bipolar) carried out by subjects. The first issue can be regarded as a partial test of the validity of the general *class* of multi-attribute models, while the second and third issues are aimed at discovering important differences between two representative for-

[1] There have been many measures of the B_{ij} term used in studying the adequacy-importance model. The particular measure used in this study is congruent with the definitions advanced by Wilkie and Pessemier [36], yet different from the adequacy measure used by Sheth and Talarzyk [31] or by Bass and Talarzyk [10]. Thus we are speaking of adequacy-importance models only in some generic sense, since the measures used differ within that class. The adequacy measure used here does differ from the likely-unlikely measure to be used in the Fishbein case below, and there is a clear difference between importance and evaluation as measures of the other component.

mulations within the general class of models. To the extent that heterogeneity of response patterns occurs for either formulation, confidence in the validity of the model as a representation of subjects' personal combination rules suffers. To the extent that subjects' implicit coding differs from researchers' coding schemes, confidence in these specific operational rules suffers, even if response patterns were homogeneous. However, the issue of heterogeneity (and hence the specific components used) seems more crucial for model validity than the coding issue. This is believed to be a much more fundamental and meaningful approach to model validation than previous verbal or correlational attempts. The specific task and its appropriateness for examining these issues are dealt with in more detail in the following section.

METHOD

Background

The rationale and analytical technique underlying this study are derived from the information integration work of Anderson and his colleagues [2, 3, 4, 5]. Anderson [2] first coined the term "cognitive algebra" to denote the algebraic models used by subjects to combine pieces of information. In work particularly related to this study, he and Shanteau [6, 28] examined subjective utility models of the form of (1) and (2). They found that subjects multiplied terms, but that adding across the resulting products did not hold exactly [6]. Tversky [34] also studied the subjective utility model and found that multiplication was upheld.

Both Anderson [4] and Birnbaum [13] have argued that factorial designs are most appropriate for assessing subjects' cognitive algebra. In this form of analysis, each term in the model is considered to be a factor in an ANOVA design. In this study there are two terms or factors for each model—I_i and B_{ij} for the adequacy-importance model and a_i and b_{ij} for the Fishbein model. Five levels were defined for each factor in both models, allowing construction of two 5 × 5 factorial designs.

The Experimental Task

Subjects were divided into two groups, one group being presented with stimuli of the adequacy-importance type, the other with stimuli of the Fishbein type. Adequacy-importance subjects rated profiles of the type shown earlier, with 2 replicates each of the 25 profiles in the complete 5 × 5 factorial design for a total of 50 profiles. All 50 profiles were given at once so that replicates were given by interspersing them within the first administration.

For the Fishbein task, each subject rated profiles of the form:
You believe that Brand X is
very likely:__:__:X:__:__: very unlikely
to possess a quality which you personally feel is:
very good:__:__:__:__:X:very bad.
To you, using Brand X would be
very favorable:__:__:__:__:__:__:__:__:__:__:__: very unfavorable.

Once again 2 replicates each of the 25 profiles in the 5 × 5 factorial design were rated.

Before rating the profiles, each group of subjects was first given a warm-up task to acquaint them with the factors utilized [4]. For the adequacy-importance subjects the warm-up task consisted of rating I_i, B_{ij}, and A_j for five attributes and seven brands of toothpaste; Fishbein subjects rated a_i, b_{ij}, and A_j for the same attributes and brands. However, the instructions for the profile rating tasks emphasized that subjects were to rate a hypothetical product, Brand X, and not toothpaste; i.e., they were asked to consider how they would use information on attributes for products in general. Also, subjects were told that they should evaluate Brand X assuming that they had information only on the single attribute in the profile, and that the two pieces of information in the profile were intended to represent their *own* feelings about the attribute, not someone else's. The response scale used for the profiles was anchored [4], by presenting subjects with "extreme" profiles first, e.g.:

Very high:X:__:__:__:__:very low
Very important:X:__:__:__:__:not important at all

Order of the other profiles was randomized. Subjects were undergraduate students enrolled in an introductory psychology course, with 85 subjects receiving the adequacy-importance stimuli and 77 subjects receiving the Fishbein stimuli. Different subjects were used for the two tasks because of the possible effects of order bias and fatigue. The factorial rating task described above differs from other rating tasks often used in consumer research. Hence, a discussion of the appropriateness of the task for the issues studied is in order.

Appropriateness of the Task

The first issue to be raised regarding the task is that multi-attribute models are not being directly studied; only a single attribute is considered. However, the results of studying a single attribute can yield insights into how subjects combine components; starting with a simpler case as a means for understanding a more complex phenomenon is a common strategy for research. One problem may arise in that subjects may *assume* values for other attributes in addition to the information on the single attribute actually given to them. If they assume that other attribute values would be like the attribute presented, no problems arise. However, if they assume other attributes are generally positive, this could cause problems. This point will be discussed further in the results section. Any sort of configural relationship among attributes cannot be studied using the present single attribute design. This is clearly a shortcoming and should be the subject of future research. Although the question of combining ratings for more than one attribute was not addressed in this study, Anderson and Shanteau [6] have shown how a four-way factoral design can be used to study models of the form $a_1 b_{11} + a_2 b_{21}$ or $I_1 B_{11} + I_2 B_{21}$, for example. Of course such a four-way design presents even more complex and numerous stimuli to the subjects. Hence it seems reasonable to examine the simpler case first.

The second issue is the conceptualization of the factorial rating task. What does the task demand of subjects, and how are they likely to react? This seemingly basic issue has not been treated anywhere in the literature on human judgment to the authors' knowledge. The most straightforward way of conceptualizing the task is that subjects are presented with a particular factorial sample from the universe of possible combinations of a_i, b_{ij} or I_i, B_{ij} components. The scales used in this study for these components have been used in previous research, and have thus been assumed to have some meaning for subjects. In addition, all of the combinations in the factorial design could potentially occur, i.e., none of them are "nonsense" combinations.

The important question is how subjects respond to the combinations, as represented in the profiles. Subjects were told to assume that the profiles referred to a hypothetical product with hypothetical attributes. Since the importance and evaluation ratings vary over profiles, it seems clear that subjects cannot see all the profiles as being related to the same attribute. What does this imply about responses? There seem to be two basic ways of conceptualizing subjects' response modes. First, subjects may intuitively combine the components in a content-free manner, without assuming particular attributes, brands, etc. This was the thrust of telling subjects that the attributes were hypothetical. The second way subjects may respond is to assume content, i.e., to imagine some particular product class and some particular attribute, and perhaps even a particular brand. If this is the case, then each subject may not even assume the same content over the course of the task. Thus a great deal of heterogeneity would be introduced. For example, some subjects may assume monotone (more is better) attributes and others may imagine non-monotone attributes. Although research on ideal point formulations [8] has appeared, there is *no theory* for how responses differ across attributes, brands, or even product classes. Research on both the Fishbein and adequacy-importance models has thus implicitly assumed homogeneity, with all attributes and brands being treated equally. Hence, regardless of what subjects assume, the homogeneity assumption implies that there should be no differences in combination rules. This is indeed a strong assumption. However, in the absence of any theory of heterogeneity in the literature, it seems reasonable. In any event, there is no reason to expect any biases caused by the assumption to affect differentially the two models. It should downgrade the performance of both.

The argument above has been concerned with showing that subjects' responses are relevant for the problem at hand regardless of any content assumed by the subjects. Another important issue is whether the responses have external validity. The factorial rating task is certainly low in mundane reality [7], and may cause consistencies and simple rules to appear because of noninvolvement. One way of claiming external validity for the task is to note whether there are individual differences in the rules subjects appear to use for combining the profile information. The more individual differences there are, the more likely it may be that the factorial task itself is not dominating responses, but that the particular components used (a_i and b_{ij} or I_i and B_{ij}) are influencing the results. A second argument in favor of the factorial approach is based on past empirical results. Anderson and his colleagues have been able to verify theoretical models in diverse areas by using the factorial task, and more importantly, the theoretical models examined have involved *different* cognitive algebra: multiplying [28], adding and averaging [4], ratio models [2], and so forth. This implies that the task does not bias the *form* of the cognitive algebra, although the task may cause an upward bias in the consistency of subjects. On the whole, therefore, although there are some problems, some degree of external validity seems present.

It has been argued that subjects' responses can be meaningfully studied to examine cognitive algebra. It remains to be shown that the responses are applicable for studying the particular models of interest, Fishbein and adequacy-importance. The technique is equally relevant to both models where the dependent measure is sôme supposed interval scale of affect [11, 14, 22, 25, 31, 35]. In this case the assumption is implicitly made by the modeler that each attribute makes comparable contributions to affect. This assumption of equal contribution to affect is *not* made in those cases where the adequacy-importance model has been used to predict preference ranking [9, 10]. In fact, if the major concern is to predict order of preference among a set of brands in a static situation, the analyses in this article are not totally relevant, since importance is *fixed* for each attribute among the set of brands. Thus all that is relevant is the relationship between B_{ij} and subjects' affect ratings as B_{ij} changes for fixed I_i. However, in attitude *change* situations where an attribute's importance rating is altered [22], the results are quite relevant. For example, assume that attribute i has low importance ($I_i = 1$), and a particular brand has a low degree of possession ($B_{ij} = 1$). Because of advertising or other factors, attribute i undergoes a change in importance to high importance ($I_i = 5$) for some segment of the population. The adequacy-importance model would predict that people for whom the importance was changed would tend to rank the brand higher than people for whom importance was not changed, assuming comparable importance and perception scores for the other attributes. This is nonintuitive; if a brand does not possess a very important attribute, it should be less desirable than a brand perceived as not possessing an unimportant attribute. Clearly the task of this study is not strictly testing for this effect, but even for the rank-order case of adequacy-importance models the task is somewhat relevant. However, since the static case is studied here, the results presented later should be viewed as mainly relevant to the case where an interval affect measure is used, as has been the case for a good deal of research on both models.

Thus the task seems relevant and valid for studying both the adequacy-importance and Fishbein models. Again, it should be emphasized that the main focus of the article is how *individuals* combine information. For that purpose, the assumptions made by model builders about appropriate scale codes for the model's components are irrelevant. However, to draw conclusions from the results about the appropriateness of a particular model, the modeler's assumptions must be examined, as done earlier. The next section addresses the question of how subjects' ratings can be analyzed to yield insight into how pieces of information are combined, i.e., the subjects' cognitive algebra.

Classification of Cognitive Algebra

Although subjects perform either an adequacy-importance task or a Fishbein task, their cognitive algebra may not correspond to that theoretically implied by the task. Thus it is important to distinguish between the type of task performed and the type of cognitive algebra employed. For example, subjects receiving the adequacy-importance stimuli might yield data plots of the form of Figure 1(c). A variety of other possibilities might also exist which could be classified and examined.

Each of the theoretical models implies a set of statistical properties and a particular graphical form for the affect ratings in the task given. These statistical properties and graphical analyses were employed to develop a number of categories into which subjects were classified. To some extent the categories were developed ex post, after the data had been collected, as it became clear that categories not suggested from the simple coding and combination rules were of considerable importance.

In analyzing the rating data, the assumption of interval scale affect measures was made. This is a crucial but necessary assumption since the unipolar multiplying model and the adding model of Figures 1(a) and 1(b) cannot be distinguished on the basis of ordinal data [21]. Green [19] has discussed uses of MONANOVA to transform data to additivity to attempt removal of interactions. Such transformations were not used for several reasons. First, one of the most crucial distinctions to be drawn is between data of the form shown in Figure 1(a) and that of Figure 1(c), and the crossover interaction of 1(c) *cannot* be removed by monotone transformation. Second, in past research using the models tested here interval scale data have been assumed for the affect measure in many cases. Third, *within-individual* analyses were performed, meaning that no interpersonal comparisons were undertaken. Finally, Anderson has argued that with the use of the anchoring and warm-up procedures discussed earlier, the success of his algebraic models lends credence to treating ratings as interval scales [4].

The basis for all of the statistical tests is analysis of variance applied to the rating data of *each individual subject*. Since there are two entries per cell, the statistical significance of both main effects and the interaction term can be tested. Note that the ANOVA approach requires only the assumption that the *dependent* variable is an interval scale. No assumptions need be made about the scaling of the independent variables. This is important, since the models examined are multiplicative models. The results obtained from such models *are not invariant* if both the components have only interval scaling and they are actually multiplied. Having both components ratio-scaled is necessary. Hence, approaches using *a priori scales* for the components have severe problems, since subjects presumably do not provide ratio-scaled ratings of such components as importance or evaluation. Thus multi-attribute models seem to require more sophisticated data than subjects can provide. However, the ANOVA approach avoids this problem since subjects are *not asked to rate the components*. Thus the patterns of responses can be examined *without* attacking this scaling issue. These patterns are strictly an empirical issue relating to subjects' *implicit* use and coding of the components, and not related to this scaling issue per se. In fact Anderson [3] uses the subjects' responses to *derive* scales rather than assume them a priori. However, to reiterate, scaling of the independent variables is not the present focus; it is unnecessary in order to *determine* the model forms discussed later. Only assumptions about the dependent variable, affect, are necessary for this purpose. Coding and scaling considerations are useful for *interpreting* these model forms, however.

As indicated previously, both statistical and graphical criteria were used to develop the categories. However, the statistical criteria which were employed varied to some extent; for certain categories it seemed appropriate to employ conventional significance levels ($\alpha = .05$), while for others the amount of explained variance as measured by $\hat{\omega}^2$ was employed [19, 20].

The particular reasons for choice of criteria will be discussed as the categories are presented. Before proceeding to the description of the categories, it seems useful to show an example of the raw data which were analyzed and the results of the ANOVA based on these data. This illustration may help to clarify some of the discussion to follow. Table 1 shows the data and the ANOVA for Subject 113, who performed the Fishbein task. In the top portion of the table are the affect ratings for each combination of evaluation and belief levels, with each entry representing the mean of the two replicates. The bottom portion of the table shows the ANOVA. These data are graphed in Figure 5(a).

Cognitive Algebra Categories

Category 1—No Significant Effects—"No Effects." Subjects falling into this category have no significant effects for either the treatments or the interaction. They either act randomly or concentrate their affect ratings at a single level. As a group they are essentially uninteresting and their ratings may result from low task involvement.

Category 2—Significant Main Effects—"Adding Model." Subjects in this category have either one or both main effects significant, but have no significant interaction effects. Their data plots correspond to Figure 1(b), i.e., a set of parallel lines. The underlying theoretical model implied is addition of terms.

Category 3—Significant Main Effects, Significant Interaction Concentrated in the Bilinear Term—"Unipolar Multiplying Model." Subjects in this category have two significant main effects and a significant interaction concentrated in the linear × linear (bilinear) component of the interaction, with a single degree of freedom [3, 6, 28]. The total sum of squares for interaction, with 16 degrees of freedom, can be partitioned into this bilinear term with 1 degree of freedom and a residual term with 15 degrees of freedom. The appropriate test, therefore, is for a significant interaction, and a nonsignificant residual interaction [19, 24, 33]. The data correspond to the plot in Figure 1(a), i.e., a diverging fan of straight lines in the upper right quadrant. The underlying theoretical model is multiplication of data coded in a unipolar fashion, which corresponds to the assumptions of the adequacy-importance model. The basic principle underlying this category is that in combining information by multiplying, holding one factor constant produces data which are a straight line in the other factor; hence, the emphasis is on the linear by linear (bilinear) interaction term. For computational details, see [4, p. 157].

Category 4—Major Portion of Explained Variance in Interaction—"Bipolar Multiplying Model." In this category the first departure is made from the use of significance levels as criteria. The rationale is as follows: for the underlying theoretical model, the bipolar multiplying model in Figure 1(c), there is an unambiguous prediction that all explained variance (as measured by $\hat{\omega}^2$) should be concentrated in the interaction. The prediction for this model contrasts with that of the unipolar multiplying model for which there is no a priori basis for assigning explained variance, except to say that it should be distributed among the treatment effects and the interaction. Whereas the use of a standard significance level is a sensible procedure for defining the category corresponding to the unipolar multiplying and adding models, the use of explained variance for the bipolar multiplying model gives additional information about the data. It is a particularly appropriate procedure in view of the nature of the data. There were cases in which both main effects were significant (in addition to the significant interaction), yet the amount of explained variance in the interaction was over 70%, while in total the two main effects accounted for less than 10%. In such cases the data plots were virtually indistinguishable from those in which there were no significant main effects, but a significant interaction. For those reasons, it seemed reasonable to develop criteria based upon explained variance.

The particular criteria which had to be met for inclusion in this category were as follows: the amount of explained variance in the interaction, as measured by $\hat{\omega}^2$, had to be at least 35%, and in addition, had to be at least 3 times greater than the explained variance for each main effect taken separately. Although these criteria are purely judgmental, an examination of the characteristics of subjects placed in this category, which will be discussed shortly, lends credence to the particular parameters chosen.

Table 1
ILLUSTRATIVE DATA AND ANALYSIS

Raw Data for Subject 113 for the Fishbein Task

Factor 2 Evaluation levels	Factor 1 Belief levels 1	2	3	4	5	Average
1	10.5	8.0	6.0	3.0	1.0	5.7
2	9.0	8.5	7.0	4.0	2.5	6.2
3	6.0	4.5	6.0	7.0	6.5	6.0
4	2.5	4.0	6.0	9.5	8.5	6.1
5	1.0	3.5	6.0	9.0	11.0	6.1
Average	5.8	5.7	6.2	6.5	5.9	6.02

Analysis of Variance Summary Table—Subject 113

Source	Sum of squares	Degrees of freedom	Mean square	F	$\hat{\omega}^2$
Belief	1.483	4	0.371	0.639	0.000
Evaluation	4.283	4	1.071	1.846	0.005
Interaction	382.717	16	23.920	41.241[a]	0.925
Error	14.50	25	0.580		
Total	402.984	49			

[a] Significant beyond $p < .001$.

Category 5—Major Portion of Explained Variance in One Main Effect and the Interaction—"Asymmetric Multiplying Model." Subjects in this category have one strong main effect which explains roughly the same proportion of the variance as does the interaction. The specific criteria formulated were that $\hat{\omega}^2$ for one main effect was less than 10% of the total $\hat{\omega}^2$, and further, was less than 25% of the $\hat{\omega}^2$ for the other main effect. Such data were not predicted from the simple coding and combination rules discussed earlier. Graphical presentation of this model and discussion of its implications are presented later.

Category 6—Significant Main Effects, Significant Interaction Not Concentrated in the Bilinear Term—"Curvilinear Multiplying Model." Subjects in this category have two significant main effects, but tests of the interaction showed a significant residual interaction after the bilinear term was extracted. This category is clearly related to Category 3. The major distinction between them is nonlinearity which is present in this model, but not in the unipolar multiplying model. In addition, the amount of explained variance in each main effect is sufficiently large relative to that in the interaction, so that subjects are not classified into Categories 4 or 5. Since data of this type were not predicted from the simple coding and combination rules discussed earlier, graphical presentation and psychological interpretation are undertaken later.

Category 7—"Unclassified." Subjects in this category have data which do not meet the criteria of any of the categories discussed. Essentially they have relatively low total explained variance, yet they also have effects which reach significance. Since they are few in number, they will not be discussed further.

These classification rules are relatively complex, but are necessary to classify fully subjects' cognitive algebra patterns. In addition, some of the rules used are clearly arbitrary. However, this is the first attempt at developing a classification scheme of this sort, and a rough categorization is thus unavoidable.

RESULTS

The results of the study are shown graphically in Figures 2 to 7 and summarized in Table 2. Two measures of reliability of subjects' ratings are presented in Table 2—$\Sigma\hat{\omega}^2$, the total variance accounted for by the ANOVA, and r, the product-moment correlation between the two sets of 25 ratings. Note in Table 2 the particularly important point that the homogeneity of response patterns, as indicated by the distribution of subjects over categories, is relatively much greater for the Fishbein task than for the adequacy-importance task. The structure of these individual differences was

Table 2
SUMMARY OF RESULTS FOR TWO RATING TASKS

Category number	*Model type*	*N*	*Adequacy/ beliefs* $\hat{\omega}^2$	*Importance/ evaluation* $\hat{\omega}^2$	*Interaction* $\hat{\omega}^2$	$\Sigma\hat{\omega}^2$	*r*
			Adequacy-Importance Task				
1	No effects	3	.031	.033	.136	.200	.231
2	Adding	19	.389	.353	.020	.761	.814
3	Unipolar Multiplying	14	.402	.267	.158	.827	.866
4	Bipolar Multiplying	11	.053	.017	.676	.746	.767
5	Asymmetric Multiplying	13	.340	.027	.381	.747	.777
6	Curvilinear Multiplying	18	.359	.263	.239	.860	.889
7	Unclassified	7	.322	.189	.170	.681	.729
	Total	85	.316	.202	.245	.762	.817
			Fishbein Task				
1	No effects	4	.000	.075	.025	.100	−.067
2	Adding	18	.237	.339	.045	.621	.687
3	Unipolar Multiplying	3	.169	.174	.228	.570	.623
4	Bipolar Multiplying	44	.021	.052	.702	.776	.831
5	Asymmetric Multiplying	4	.037	.261	.389	.687	.695
6	Curvilinear Multiplying	2	.276	.238	.401	.915	.923
7	Unclassified	2	.064	.293	.171	.527	.546
	Total	77	.085	.147	.457	.689	.764

certainly not predicted, and will be considered later.

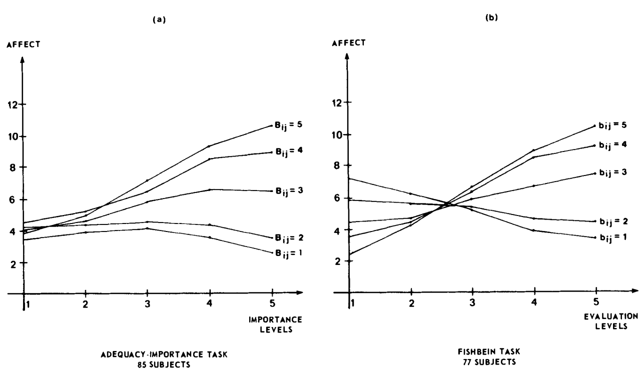
One further issue should be discussed concerning graphical presentation of the rating data. Anderson [3, 4, 6] uses what he calls functional measurement to obtain subjective scale values for the stimulus factor levels which are then used to aid in plotting the data. The details are given in Anderson and Shanteau [6]; the basic idea is that the row and column averages of the subject's rating data provide scale values for the levels of each factor (see [27] for a dissenting view). If the factor used for the horizontal axis in plotting the data is spaced on the horizontal axis according to these functional scales, the form of the model becomes more apparent. However, this is true only for those cases which satisfy the adding or unipolar multiplying model. Departures from the model infirm the scale values. Thus, to provide comparability across data plots, all graphs were plotted as if the levels of the independent variables (factors) were intervally spaced from 1 to 5 on the horizontal axis. These are only labels for the graphs, however. As discussed above, specific scale values for the independent variables need not be assumed in order to analyze the data.

Figure 2 graphs the summary data from the two rating tasks. The data from the adequacy-importance task exhibit the diverging fan pattern, which is characteristic of the unipolar multiplying model's algebra, although there appears to be a distinct nonlinearity in the data, as evidenced by its curvilinear nature for low levels of B_{ij}. The data from the Fishbein task clearly exhibit the bipolar multiplying effect required by the Fishbein model, although some asymmetry is present. Based on the results of this aggregate level analysis, it might be concluded that both the adequacy-importance model and the Fishbein model are reasonably valid representations of consumer decision rules. In both cases, the multiplicative form of the multi-attribute model appears to have been supported. However, the intent of this research is to examine behavior at the individual level, since heterogeneity of response patterns is an important criterion for model validity; accordingly, the various categories identified in the data will now be discussed.

Figure 3 shows the data of subjects who satisfied the criteria for an adding model. The plot in 3(a) is for a typical subject, while 3(b) and 3(c) are the grouped data for subjects who received the adequacy-importance and Fishbein tasks, respectively. Both of the summary plots correspond well to the theoretical plot of Figure 1(b), and a comparable number of subjects from each task employed this model, 19 for the adequacy-importance task and 18 for the Fishbein task.

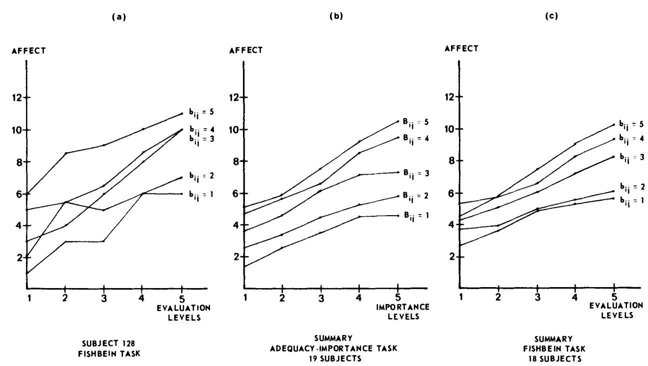
The graphs in Figure 4 represent the data of subjects who satisfied the criteria for the unipolar multiplying model. Data for a typical subject are shown in 4(a), while 4(b) shows grouped data for the 19 adequacy-importance task subjects. Both graphs correspond well to the theoretical plot of Figure 1(a), although there is a curvilinear tendency in the grouped data. Only three subjects from the Fishbein task satisfied the criteria for this model, Figure 4(c).

Figure 2
GROUP SUMMARIES OF AFFECT RATINGS[a]



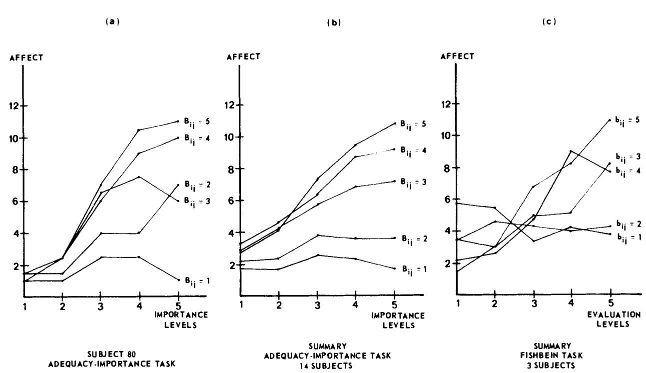
[a]The B_{ij} and b_{ij} labels represent factor levels, not scale values.

Figure 3
ADDING MODELS SUMMARY[a]



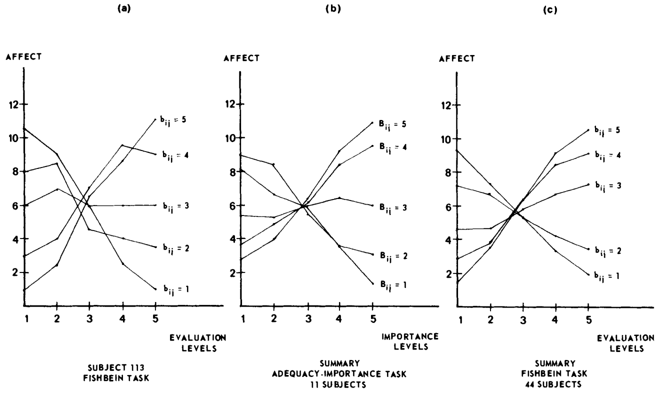
[a]The B_{ij} and b_{ij} labels represent factor levels, not scale values.

Figure 4
UNIPOLAR MULTIPLYING MODELS SUMMARY[a]



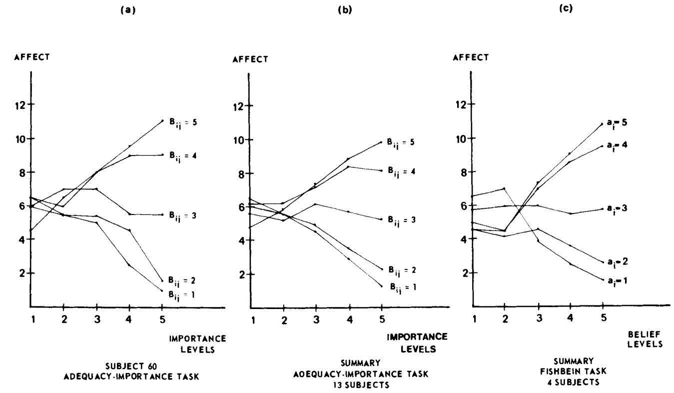
[a]The B_{ij} and b_{ij} labels represent factor levels, not scale values.

Figure 5
BIPOLAR MULTIPLYING MODELS SUMMARY[a]



[a]The B_{ij} and b_{ij} labels represent factor levels, not scale values.

Figure 6
ASSYMETRIC MULTIPLYING MODELS SUMMARY[a]



[a]The B_{ij} and b_{ij} labels represent factor levels, not scale values.

The graphs in Figure 5 are data for subjects who satisfied the criteria for the bipolar multiplying model. A typical subject data plot is shown in 5(a), while 5(b) and 5(c) show data for 11 adequacy-importance subjects and 44 Fishbein subjects respectively. All plots correspond well to the theoretical graph of Figure 1(c). Table 2 indicates the high proportion of explained variance attributable to the interaction relative to the main effects.

The results above show that only 14 subjects (17%) followed the underlying theoretical model for the adequacy-importance task, while 44 subjects (57%) appeared to utilize a bipolar multiplying model for the Fishbein task. This difference is highly significant (χ^2 = 29.1, d.f. = 1, $p < .001$). Even under the very conservative assumption of ordinal equivalence, in which case the adding models of Category 2 are combined with the unipolar multiplying models for the adequacy-importance task, the Fishbein model still performs significantly better (χ^2 = 5.4, d.f. = 1, $p < .05$). Further, for the adequacy-importance task 11 subjects acted as though they were using a bipolar multiplying model, while for the Fishbein task only 3 subjects appeared to be following the unipolar multiplying model. Thus, not only is there relatively more heterogeneity for the adequacy-importance model, but the coding implied by subjects' responses differs from that of the model, as it is typically operationalized.

Figure 6 shows the data for subjects satisfying the criteria of the asymmetric model, 6(a) being for a typical subject, while 6(b) and 6(c) are grouped data for the two tasks. Fourteen subjects from the adequacy-importance task and four from the Fishbein task are involved. It should be noted that for these plots the horizontal axis corresponds to the stimulus scale for which the main effect was very small (importance for the adequacy-importance task and belief for the Fishbein task—see Table 2). The plots form diverging fans of straight lines and are symmetrical about a line drawn parallel to the X-axis. If the scales are reversed, the plots look quite different, forming a narrow crossing pattern running from the bottom left to top right of the graph.

This particular category is interesting for two reasons. First the main effect which accounts for high explained variance differs for the two tasks. The adequacy component demonstrates a strong main effect for the adequacy-importance task, while the evaluation component exhibited the strong effect for the Fishbein task. (Within each task all subjects in this category had the same strong main effect.) The former result is particularly interesting in light of previous research on a hybrid of the adequacy-importance model advocated by Sheth [29], which has been labeled the "beliefs-only" model. Some studies have reported rather substantial correlations for this model [16, 23], so it seems more than mere coincidence that the adequacy factor, rather than the importance factor, was the one which exhibited a main effect in the Asymmetric Multiplying group.

A second observation is that the pattern of data shown in Figures 6(b) and 6(c) is exactly what would be expected if subjects had coded one of the two types of information on a bipolar basis and the other as unipolar before "multiplying." In the figure, the horizontal axis corresponds to the dimension which was apparently treated in a unipolar fashion. Thus, it appears that the 13 adequacy-importance task subjects falling into this group treated importance as unipolar, while the 4 Fishbein task subjects treated likelihood on a unipolar basis. These results, while characterizing only a relatively small portion of the total sample, may serve to point out sources of ambiguity in the two models as they are currently operationalized. There are alternative interpretations of this pattern of results: for instance, subjects may have "coded" both scales from -1 to $+3$, instead of -2 to $+2$ as the Fishbein model assumes, or 1 to 5, as the adequacy-importance model assumes; or subjects assumed when given an unimportant attribute that the brand was moderately acceptable on other attributes not presented and took this into account in their ratings. However, the interpretation given is more favorable to the adequacy-importance model, and hence more conservative given the conclusions, since it assumed only one scale is treated as bipolar rather than both.

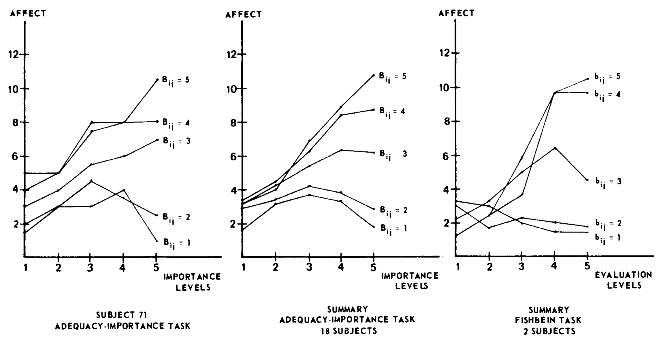
It is now fairly clear that a relatively large proportion of the subjects in the adequacy-importance task treated the adequacy stimuli as bipolar rather than unipolar. Eleven subjects in the bipolar multiplying category and 13 more in the asymmetric multiplying category fit this description, meaning that over 40% of the subjects treated the adequacy data differently than the typical operationalization of the adequacy-importance model.

Also, considerable controversy has surrounded the importance component of the adequacy-importance model [28]. Present results indicate that the mixed findings related to this component may be traced to individual differences in how subjects treat a scale ranging from "not important at all" to "very important." While some may view the scale as intended by the researcher, other subjects appear to treat the scale as something akin to a scale whose end points should read "very important (not to have the attribute)" to "very important (to have the attribute)." The latter scale is clearly bipolar in nature, while the former is unipolar. The fact that there appears to be some ambiguity in how subjects interpret the importance scale suggests further work on the development of a reliable and valid measure of this component is necessary. This component very probably led to the greater heterogeneity of response patterns for the adequacy-importance model.

Finally, analogous to the above arguments, there is some indication that subjects have difficulty interpreting the "likely-unlikely" scale used to operationalize the b_{ij} component in the Fishbein model. This finding is in agreement with previous research [22], and Ahtola [1] has argued that to treat subjective probabilities (which is what b_{ij} represents theoretically) as bipolar is inappropriate. The fact that the overwhelming majority of subjects in the Fishbein task tended to treat the likelihood measure as bipolar is somewhat surprising, since intuitively likelihood would seem to be a unipolar construct. One plausible explanation for this phenomenon is that subjects were engaging in cognitive balancing, rather than cognitive adding, which is the basis for the Fishbein model [1]. However, the test this hypothesis would require an attitude change paradigm, which was beyond the scope of this research.

Finally, Figure 7 shows the data for subjects satisfying the criteria of the curvilinear multiplying model, 7(a) being for a typical subject, and 7(b) for grouped data for the 18 adequacy-importance subjects. Only two Fishbein subjects satisfied the criteria. The plots show, as expected, some relationship to those of Figure 4, the unipolar multiplying model. The suggestion of a diverging fan of straight lines is present, but there is a marked curvilinearity at low levels of adequacy (B_{ij}). In all cases the data show that at low importance levels, affect was either low and constant, or low and increasing as belief increased; but, at low levels of belief, the affect value was initially low, rose, and then fell again as importance increased. The suggestion of this pattern is present even in the summary of the unipolar multiplying model, Figure 4(b).

Figure 7
CURVILINEAR MULTIPLYING MODELS SUMMARY[a]



[a]The B_{ij} and b_{ij} labels represent factor levels, not scale values.

The reason for such results under the adequacy-importance task becomes clear upon examination of the algebraic assumptions underlying the model. The adequacy-importance model assumes that an attribute for which belief of possession is *very low* (B_{ij} = 1) but is *very important* (I_i = 5) will contribute *more* to affect than an attribute for which belief of possession is *very low* (B_{ij} = 1) but which is *not important at all* (I_i = 1), since $(5)(1) > (1)(1)$. (Again, note that these numbers are used as examples of typical scale values used in previous research, *not* as scale values assumed for this study. Scale values need not be assumed for the ANOVA.) This prediction is nonintuitive to say the least, and subjects in this category reacted in a way counter to this model algebra assumption. Again, recall that this argument is not strictly applicable to the cases where the adequacy-importance model has been used for ranking alternatives rather than predicting an affect rating.

The presence of the curvilinear effect in Figure 4(b) for the unipolar multiplying model suggests that the adequacy-importance model had this problem even for subjects who fulfilled the statistical criteria for the adequacy-importance model's cognitive algebra. Thus the curvilinear category contains subjects who essentially multiply but who show even more marked deviations from linearity at low levels of belief than the unipolar multiplying subjects.

The total set of results is complex, so the implications are now summarized. One of the purposes of the present research was to examine the validity of the multiplicative assumptions underlying the general class of multi-attribute attitude models by using a simplified single-attribute task. From Table 2 it can be seen that a relatively large number of the subjects in both tasks (66% for the adequacy-importance task and 69% for the Fishbein task) were essentially behaving as though they multiplied the 2 components of the model. This finding can be regarded as strong support for the validity of the general class of models, insofar as the single-attribute task yields information about the multi-attribute case.

A second purpose was to examine the differences between the components and coding used for the Fishbein model and the adequacy-importance model in terms of (1) the heterogeneity of subjects' response patterns, and (2) the data coding procedures subjects apparently used in the task corresponding to each model. While there were virtually the same number of "multipliers" among the subjects engaging in the respective tasks, there is clear evidence in Table 2 that the cognitive algebra pattern of the Fishbein task subjects was much more homogeneous than that of the adequacy-importance task subjects. Also, the typical coding rules of the Fishbein model seem more representative of subjects' implicit coding rules than those of the adequacy-importance model. Of the subjects classified into the 4 multiplying categories, only 25% fit the assumed model (unipolar multiplying) under the adequacy-importance task, while 83% fell into the appropriate category (bipolar multiplying) in the Fishbein task. Furthermore, nearly 25% of the adequacy-importance task subjects were classified as bipolar multiplying, but only 3 subjects in the Fishbein task fit the unipolar multiplying category.

From this pattern of results, it is clear that the type of cognitive algebra a person appears to use is affected substantially by the scales used to measure cognitive constructs. Unfortunately, it is impossible to know which way of representing cognitive constructs corresponds with "truth." Therefore, reliance must be placed on external criteria such as homogeneity of response patterns across subjects and the ultimate usefulness of the model, as operationalized, to the policy-maker. Based on these criteria, the Fishbein form of the model seems relatively superior to the adequacy-importance model. This conclusion is based on the more homogeneous pattern of responses exhibited by subjects in the Fishbein task. Of course there is still heterogeneity within each task, so these conclusions are only relative, not absolute. In addition, the subjective coding apparently used by subjects corresponds well with the coding typically used in the data analysis stage for the Fishbein task. Thus the marketing manager can utilize the Fishbein model with considerably more confidence than he can the adequacy-importance model where he wishes to predict affect. If he wishes to predict only the rank order of brands, the results do not speak directly to the issue; however, it may be argued, as above, that these results should give cause for thought even in the latter case.

DISCUSSION AND CONCLUSIONS

The results indicate individual differences in subjects' combination rules. This implies that the task does have some degree of realism, since different rules were shown. However, future research should more directly examine this issue. Lack of involvement in the task may lead to simple additive rules; this could be checked by attempting to manipulate degree of involvement and examining the resulting cognitive algebra types. Also, one could attempt to study whether the factorial task forced consistency in subjects' responses by interrupting the task and determining the effect of the interruption on the resultant cognitive algebra types.

An important advantage of the ANOVA method is that it allows the study of individual differences in handling information not easily undertaken in the typical correlational design. The study of individual differences reported here is one contribution of this study to the methodology of information integration research. This issue has been essentially ignored by Anderson [4], as his work has usually involved small numbers of subjects. Individual differences analysis could be an important tool for examining issues such as whether subjects combine the information for all attributes in the same fashion; this was simply assumed earlier, since there are no relevant data or theory. It may be true that subjects use different combination rules for different subsets of attributes. This seems to be a fruitful area for further research. One additional contribution to the methodology of information integration studies is in the use of $\hat{\omega}^2$ as a criterion for selecting model types. It is believed that this study is the first to use $\hat{\omega}^2$ in this manner.

Finally, there are significant results for attitude research. Again, these results are subject to the caveats that only single attributes were used and that the results are relevant more for models attempting to predict affect than for models predicting rank preferences. Given these limitations, the main findings are twofold: (1) there is considerable support for the multiplicative form of multi-attribute models, although there is a significant proportion of the population which appears to use some other form of data handling strategy; and (2) the Fishbein model represents the cognitive algebra of subjects relatively better than the adequacy-importance model since there is greater homogeneity of response in the Fishbein task. The evidence also points strongly to the conclusion that the cognitive components of the Fishbein model (i.e., a_i and b_{ij}) are indeed treated in a bipolar manner and multiplied, as assumed by the Fishbein model algebra.

For the adequacy-importance model the evidence points to the conclusion that the scales used to operationalize the model are ambiguous. Even those subjects who appeared to treat both stimuli as unipolar exhibited curvilinearity in their data. As indicated in the previous section, their behavior, while not following model algebra, is nonetheless intuitively reasonable.

The weight of the evidence, then, seems to indicate that the Fishbein components are less ambiguous than the adequacy-importance model components, and that the Fishbein model algebra corresponds to subjects' cognitive algebra more closely than adequacy-importance model algebra. The adequacy importance model, with its typical unipolar coding scheme, makes very nonintuitive assumptions about high importance-low belief items, and the nature of importance itself as a unipolar rather than bipolar scale seems to cause problems. Most subjects were able to deal with "very bad" in the Fishbein task, but many subjects seemed uncertain about how to deal with "not at all important" in the adequacy-importance task. Thus, in terms of an approach to validation using individual subject patterns of information combination as the data, the Fishbein approach to attitudes seems to be relatively more valid. Cohen, Fishbein, and Ahtola [16] have argued conceptually why this is so, and the data of the study presented here seem to bear out their arguments.

This finding is particularly important for marketing applications of these models, because the real issue is that subjects respond much more heterogeneously to the adequacy-importance task than to the Fishbein task. If cross-sectional analyses are to be used in real applications, then presumably one should try to minimize any controllable sources of heterogeneity. The results presented here imply that one should use the Fishbein approach to do this. This could be particularly important if the model is being used in a disaggregated sense for diagnosis.

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