Prospective Gain-Loss Utility: Ordered versus Separated Comparison

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Abstract

Koszegi and Rabin (2006, 2007) develop a model of expectations-based reference-dependent preferences, in which the agent experiences prospect-theory inspired "gain-loss utility" by comparing his actual consumption to all his previously expected consumption outcomes. Koszegi and Rabin (2009) generalize the static model to a dynamic setting by assuming that the agent experiences both contemporaneous gain-loss utility over present consumption and prospective gain-loss utility over changes in expectations about future consumption. Moreover, the authors generalize the outcome-wise "static comparison" of gain-loss utility to a percentile-wise "ordered comparison," in which the agent compares consumption outcomes at each percentile. This paper generalizes the static comparison slightly differently to, what I call, a "separated comparison." Under the separated comparison, the agent compares each consumption outcome but experiences gain-loss utility only over uncertainty that has been realized, by considering it separately from remaining future uncertainty. Effectively, the separated comparison modifies the static comparison by considering potential non-independence of the prior and updated expectations about future consumption. Thus, it reduces to the static comparison for independent prior and updated expectations and is zero if these happen to be the same. Moreover, it yields simple, tractable, and well-behaved equilibria in a wide class of economic models, because it preserves an outcome-wise nature, which makes it linear and dynamically similar to contemporaneous gain-loss utility.

Keywords: Expectations-based reference dependence, prospective gain-loss utility

JEL Codes: D03, D11

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1 Introduction

Inspired by prospect theory[1], Koszegi and Rabin (2006, 2007) introduce a generally applicable model of reference-dependent preferences. These preferences' reference point is determined by the agent's beliefs about consumption; these beliefs correspond to the fully probabilistic distribution functions conditional on the previous period's information set. Koszegi and Rabin (2009) develop a dynamic model of expectations-based reference-dependent preferences. In the dynamic model, the agent experiences contemporaneous gain-loss utility as in the static model: he compares the realization of his contemporaneous consumption with his prior beliefs about contemporaneous consumption. Additionally, the agent experiences prospective gain-loss utility: he compares his updated beliefs about future consumption with his prior beliefs about future consumption. Koszegi and Rabin (2006, 2007) formalize gain-loss utility as an outcome-wise "static comparison," under which the agent compares each consumption outcome under his updated beliefs to each consumption outcome under his prior beliefs. Koszegi and Rabin (2009) generalize the static comparison to a percentile-wise "ordered comparison," under which the agent compares his consumption under the updated beliefs at each percentile to his consumption under his prior beliefs.

Koszegi and Rabin (2009) argue in favor of the ordered comparison because, under the static comparison, the agent would experience adverse gain-loss utility if he were not to update his beliefs. More precisely, if the agent's prior and updated beliefs happen to be the same distributions, then the ordered comparison yields zero gain-loss utility, whereas the static comparison yields adverse gain-loss utility, which seems unrealistic. However, I show that the static comparison would yield zero gain-loss utility if the agent would take into account that prior and updated beliefs about future consumption are perfectly correlated. If the agent takes the correlation structure of beliefs into account, he only experiences gain-loss utility over uncertainty that has been realized in that period. Thus, the agent separates uncertainty realized in the contemporaneous period from uncertainty to be realized in future periods, and hence, I refer to this new comparison as the "separated comparison."

The ordered and separated comparisons are equivalent for contemporaneous gain-loss utility. For prospective gain-loss utility, they are equivalent when the agent's updated beliefs are degenerate, the agent is hit by a surprise shock, or the agent's beliefs are characterized by a binary distribution. However, the two comparisons are quantitatively slightly different for continuously distributed prior and updated beliefs. It turns out that these small quantitative differences of the separated comparison simplify the equilibrium solution of a wide class of

[1] Prospect theory (Kahneman and Tversky (1979)) states that people care about gains and losses relative to a reference point, whereby small losses hurt more than small gains give pleasure; that is, people are loss averse.

models. Moreover, because the separated comparison is a linear operator, the models become more tractable. Finally, as I will discuss, the separated comparison has a different, but compelling, psychological intuition. Let me briefly explain why the separated comparison simplifies the equilibrium.

The gain-loss formulation of Koszegi and Rabin (2006, 2007) has an analytical advantage over classical prospect theory in stochastic models that feature a continuously distributed consumption function. In such models, the stochastic reference point corresponds to a continuous distribution function, which makes the agent's maximization problem continuously differentiable and globally concave, if his consumption utility is a concave function. In contrast, the reference point is deterministic in classical prospect theory, which introduces a non-differentiable kink in the utility function. Such a comparison to a deterministic reference point also results from the ordered comparison when future uncertainty becomes large relative to present uncertainty, which is often the case in economic models. If the agent's updated distribution over future consumption is slightly to the right of his prior distribution, he is happy, but if it is slightly to the left, he is twice as sad. Thus, the ordered comparison causes an approximate kink in prospective gain-loss utility. In contrast, even in such environments, the separated comparison implies that the agent's degree of happiness resembles the comparison to a stochastic reference point, that is, his happiness increases smoothly when his updated distribution shifts to the right. Because contemporaneous gain-loss utility is typically determined by a related stochastic reference point, the equilibrium tends to be better behaved.

One example for such "better behavior" is that the agent's consumption is much less likely to be decreasing in the realization of some shock to his resources. A decreasing consumption function is problematic, because it cannot be an optimal solution. The agent unnecessarily experiences gain-loss utility over the decreasing part of consumption, which is painful on average as he is loss averse. Instead, he could choose a flat consumption function over this range of shock realizations. However, flat consumption implies flat beliefs, which bring about non-differentiabilities in the agent's maximization problem making the first-order condition inapplicable. Such decreasing consumption is much more likely under the ordered comparison, as the kink in prospective gain-loss utility makes the agent suddenly care much more about the future. In contrast, consumption is likely to be increasing throughout the shock's realization under the separated comparison, as prospective gain-loss utility increases more smoothly and in line with contemporaneous gain-loss utility.[2]

[2]Moreover, I argue that, in situations in which the mean of a distribution realizes, zero prospective gain-loss utility is not necessarily more compelling in the context of loss aversion than slightly adverse prospective gain-loss utility, as contemporaneous gain-loss utility would be slightly adverse too.

Beyond this simpler solution in a large class of economic models, the separated comparison reduces to the static comparison for independent distributions and yields zero gain-loss utility if prior and updated beliefs are the same. The separated comparison's disadvantage is that it requires that the correlation structure of prior and updated beliefs can be explicitly described, and it is thus less generally applicable. More precisely, the separated comparison is defined over the joint distribution function of the agent's updated and prior beliefs, whereas the ordered comparison is defined directly over the updated and prior beliefs not requiring knowledge of the joint distribution function. Nevertheless, in a large class of dynamic and stochastic models, the agent's consumption and beliefs about consumption in any future period is some function of random variables that realize in that period. In such models, the correlation structure of updated and prior beliefs can be described very easily and the separated comparison simply implies that the agent only experiences gain-loss utility over uncertainty that has been realized.

The static model of Koszegi and Rabin (2006, 2007) has been assumed in a variety of microeconomic models.[3] Moreover, the preferences have been shown to be consistent with behavioral and experimental evidence in a variety of settings.[4] Despite the popularity of the static model, no attempts have been made to incorporate the dynamic preferences into economic models potentially due to the complexity of the ordered comparison.[5] Thus, I hope that the separated comparison will facilitate the application of the preferences in dynamic and stochastic settings, as it yields simple and well-behaved closed-form solutions in a wide class of economic models.

The paper is organized as follows. First, I define the separated comparison in Section 2 and show that it reduces to the one introduced by Koszegi and Rabin (2006, 2007) for independent lotteries and yields zero gain-loss if no update in information occurs. In Section 3, I compare the separated and ordered comparisons starting with a simple example in Section

[3]Heidhues and Koszegi (2008, 2010), Herweg and Mierendorff (2012), and Rosato (2012) explore the implications for consumer pricing. Herweg, Müller, and Weinschenk (2010) do so for principal-agent contracts, and Eisenhuth (2012) does so for mechanism design.

[4]An incomplete list of papers providing direct evidence for Koszegi and Rabin (2006, 2007) preferences is Sprenger (2010) on the implications of stochastic reference points, Karle, Kirchsteiger, and Peitz (2011) on consumer pricing, Abeler, Falk, Goette, and Huffman (2012) on labor supply, Gill and Prowse (2012) on real-effort tournaments, Meng (2010) on the disposition effect, and Ericson and Fuster (2010) on the endowment effect. Barseghyan, Molinari, Donoghue, and Teitelbaum (2010) structurally estimate a model of insurance-deductible choice. Suggestive evidence is provided by Crawford and Meng (2009) on labor supply, Pope and Schweitzer (2011) on golf players' performance, and Sydnor (2010) on deductible choice. Moreover, the numerous conflicting papers on the endowment effect can be reconciled with the notion of expectations determining the reference point.

[5]Under the separated comparison, Pagel (2012a,b) incorporates the preferences into a fully dynamic consumption-savings model and a canonical asset-pricing model, respectively. Both of these models can be solved in closed form.

3.1. I move on to independent prior and updated beliefs about future consumption in Section 3.2. The separated and ordered comparisons are equivalent if prior and updated beliefs do not overlap. But, the two comparisons are very different if prior and updated beliefs happen to be the same. However, I argue that prior and updated beliefs about future consumption are not fully independent in most economic models. Updated beliefs about future consumption are fully independent from prior beliefs only if the updated distribution was expected with zero probability. In contrast, if the agent expected to learn that his updated beliefs about consumption are a finite or infinite set of possible distributions, then his updated beliefs cannot be independent from his prior beliefs.[6] Thus, in most economic models, prior and updated beliefs about future consumption are non-independent distribution functions. I compare the separated and ordered comparisons for such non-independent beliefs in Section 3.3. I find that the two comparisons are rather similar for extreme realizations of the uncertainty resolved today and if only little uncertainty remains in the future. However, the two are different in situations in which future uncertainty is large relative to present uncertainty. I show that, in such situations, the ordered comparison introduces a kink in prospective gain-loss utility, which, as explained above, complicates the equilibrium-finding process. I illustrate this result in a canonical Lucas-tree economy and argue that the psychology underlying the ordered comparison is not necessary more compelling in the context of loss aversion. Then, I illustrate another toy model about responses to signals in Section 3.4 and again argue that the separated comparison has more reasonable implications.

2 Definition of the Separated Comparison

In the following, I define the separated comparison after introducing the static comparison.

The static comparison. Koszegi and Rabin (2006, 2007) define the "static comparison" of gain-loss utility for the stochastic consumption outcome F that is evaluated relative to the stochastic reference point G as follows.

$$\int\int \mu(u(c)-u(r))dG(r)dF(c) = \int(\int \mu(u(c)-u(r))f_G(r)dr)f_F(c)dc.$$

The density functions of F and G are denoted by f_F and f_G. Consumption utility $u(\cdot)$ corresponds to the standard model of utility and depends on the level of consumption only. Gain-loss utility is determined by the piecewise linear value function $\mu(\cdot)$ with slope η and

[6] This argument is true within the equilibrium of most economic models. Nevertheless, off-equilibrium paths have to considered in the equilibrium-finding process on which the consideration of independent beliefs might be more important.

a coefficient of loss aversion λ, i.e., $\mu(x) = \eta x$ for $x > 0$ and $\mu(x) = \eta\lambda x$ for $x \leq 0$. The parameter $\eta > 0$ weights the gain-loss utility component relative to the consumption utility component, and the parameter $\lambda > 1$ implies that losses are weighed more heavily than gains, the agent is loss averse.

The separated comparison. Now suppose that F and G are non-independent distributions so that there exists a joint distribution function $F_{F,G}(c,r) \neq F(c)G(r)$ and a conditional distribution function $F_{G|F}(r|c) \neq G(r)$, with densities denoted by $f_{F,G}$ and $f_{G|F}$. However, the static formulation says that gain-loss utility is not affected by potential non-independence of F and G. If I would apply the above formulation under the assumption that F and G are independent, I would assume that the agent experiences gain-loss utility over F, which represents uncertainty that has not been realized. In contrast, if I explicitly note the dependence of F and G, I assume that the agent only experiences gain-loss utility over uncertainty that has been realized. As the new comparison assumes that the agent separates uncertainty that has been realized from future uncertainty, I call it the separated comparison.

Definition 1. The separated comparison is defined as follows

$$\int\int \mu(u(c) - u(r))dF_{G|F}(r|c)dF(c) = \int(\int \mu(u(c) - u(r))f_{G|F}(r|c)dr)f_F(c)dc$$

$$= \int\int \mu(u(c) - u(r))f_{F,G}(c,r)drdc.$$

Three points are worth noting. First, the separated comparison reduces to the static comparison if F and G are independent as $F_{G|F}(r|c) = F_G(r)$, i.e.,

$$\int(\int \mu(u(c) - u(r))f_G(r)dr)f_F(c)dc = \int\int \mu(u(c) - u(r))dG(r)dF(c).$$

Second, the separated comparison yields zero gain-loss utility if there is no update in information as $F = G$ and $F_{G|F}(r|c) = F(c)$, i.e.,

$$\int\int \mu(u(c) - u(r))dF_{G|F}(r|c)dF(c) = \int\int \mu(u(c) - u(c))dF(c)dF(c) = 0.$$

Third, if F is a degenerate lottery, as the case for contemporaneous gain-loss utility, there is no uncertainty left which has to be excluded from the comparison. Thus F and G are independent such that $F_{G|F}(r|c) = G(r)$ and prospective gain-loss utility corresponds to contemporaneous gain-loss utility

$$\int \mu(u(c) - u(r))dG(r).$$

The above definition can be rephrased in terms of random variables and Bayesian updating. Slightly abusing notation, I now suppose that prior beliefs about consumption in some future period are characterized by a random variable X with distribution function F_X. Moreover, updated beliefs about consumption in that future period are characterized by a random variable Y with distribution function F_Y. X and Y are not independent such that there exists a joint distribution function $F_{X,Y}$ and a marginal distribution function $F_{X|Y}$. Prospective gain-loss utility as defined in definition 1 is given by

$$\int (\int \mu(u(y) - u(x)) f_{X|Y}(y|x)dy) f_X(x)dx = \int \int \mu(u(x) - u(y)) f_{X,Y}(x, y)dxdy.$$

Now, the random variable Y is a Bayesian update of X according to Bayes' rule $f_{X|Y} = \frac{f_{Y|X} f_X}{f_Y}$ thus $f_{X,Y} = f_{Y|X} f_X$ and one can go back and forth between this formula and definition 1.

The correlation structure. Applying the above definition requires an explicit description of the correlation structure of G and F or X and Y respectively. Therefore, in the following I give a general example that specifies a correlation structure. Suppose that the agent's prior beliefs G about consumption in some future period consists of a finite or infinite set of possible updated distributions, which are denoted $\{F_1, ..., F_n\}$ with $n \in [1, \infty)$ with corresponding random variables $\{X_1, ..., X_n\}$ each considered with a certain probability $\{p_1, ..., p_n\}$. The agent's prior beliefs are then given by the random variable $\tilde{X} = g_{\tilde{X}}(X_1, ..., X_n)$ characterized by the distribution function $G = g_G(F_1, ..., F_n) = \sum_{i=0}^{n} p_i F_i$ if n is finite and $G = g_G(F_1, ..., F_n) = \int_i p_i F_i di$ if n is infinite, such that $G(\tilde{X}) = G(g_{\tilde{X}}(X_1, ..., X_n)) = g_G(F_1(X_1), ..., F_n(X_n))$. Now, suppose that the updated distribution $F = F_i$ realizes. Then, the distribution of G conditional on the realization of X_i denoted by x_i is given by

$$F_{G|F}(\tilde{X}|x_i) = F_{G|F}(g_{\tilde{X}}(X_1, ..., X_{i-1}, x_i, X_{i+1}, ..., X_n))$$

$$= g_G(F_1(X_1), ..., F_{i-1}(X_{i-1}), F_i(x_i), F_{i+1}(X_{i+1}), .., F_n(X_n))$$

and gain-loss utility is defined as above

$$\int \int \mu(u(c) - u(r)) dF_{G|F}(r|c) dF(c)$$

which can be rewritten as

$$\int \int \mu(u(x_i) - u(g_{\tilde{X}}(X_1, ..., X_{i-1}, x_i, X_{i+1}, ..., X_n))) dF_{G|F}(g_{\tilde{X}}(X_1, ..., X_{i-1}, x_i, X_{i+1}, ..., X_n)) dF_i(x_i)$$

$$= \int \int \mu(u(x_i) - u(g_{\tilde{X}}(X_1, .., x_i, X_{i+1}, ..., X_n)))dg_G(F_1(X_1), ..., F_i(x_i), F_{i+1}(X_{i+1}), .., F_n(X_n))dF_i(x_i)$$

$$= \int ... \int \mu(u(x_i) - u(g_{\tilde{X}}(x_1, ..., x_{i-1}, x_i, x_{i+1}, ..., x_n)))dF_1(x_1)....dF_{i-1}(x_{i-1})dF_{i+1}(x_{i+1})...dF_n(x_n)dF_i(x_i).$$

3 Comparing the Separated and Ordered Comparisons

3.1 A simple example to start with

I start with a simple example to illustrate the static, ordered, and separated comparisons. Suppose the agent lives for two periods. In period 2, he consumes a continuously distributed quantity $C_2 \sim F_2$ and his consumption utility $u(\cdot)$ is concave. In period 1, he does not consume and does not expect the arrival of any information about period 2 consumption. Thus, in the beginning of period 1, the agent's rational beliefs over period 2 consumption are given by F_2. Suppose that the agent enters period 1 and is surprised to learn that he will have 1 unit more consumption in period 2 independent of the realization of C_2. First, under the static comparison, the agent would experience prospective gain-loss utility in period 1 over period 2 consumption given by

$$\int \int \mu(u(c+1) - u(r))dF_2(r)dF_2(c).$$

Here, the agent would experience gain-loss utility not only over the one unit increase of consumption in period 2, but he would also experience gain-loss utility over the uncertainty contained in $F_2(c)$ and $F_2(r)$, which is negative on average because the agent is loss averse. Second, under the ordered comparison, prospective gain-loss utility is given by

$$\int_0^1 \mu(u(C_{F_2}(p) + 1) - u(C_{F_2}(p)))dp$$

with $C_{F_2}(p)$ the consumption level at percentile p, defined implicitly by $F_2(C_{F_2}(p)) \geq p$ and $F_2(C) < p$ for all $C < C_{F_2}(p)$. Here, the agent experiences gain-loss utility only over the one unit increase in consumption. Suppose that $u(\cdot)$ is approximately linear and $\mu(c-r)$ being a piecewise linear value function equal to $c-r$ if $c > r$ and $\lambda(c-r)$ if $c < r$, then prospective gain-loss under the ordered comparison will be approximately 1. Third, under the separated comparison, prospective gain-loss utility is given by

$$\int \mu(u(c+1) - u(c))dF_2(c).$$

Here, the agent applies the static comparison but takes explicitly into account that $F_2(c)$ and $F_2(r)$ are correlated distribution functions, i.e., $r|c = c$. Then, if $u(\cdot)$ is approximately linear and $\mu(\cdot)$ a piecewise linear value function as above, prospective gain-loss utility under the separated comparison will be approximately 1.

Although the separated and ordered comparisons are equivalent in this binary shock example, it can be easily seen that, if the agent would be surprised by a more complicated shock, the separated and ordered comparisons can be quantitatively different.[7] In Section 3.2, I illustrate these quantitative differences for independent prior and updated beliefs. But, as beliefs are not independent in most economic models, I move on to such non-independent beliefs in Section 3.3. I show that the differences between the ordered and separated comparison are most striking if the remaining uncertainty contained in F_2 is large relative to the update in information. As the separated comparison preserves the outcome-wise nature of the static comparison, it will be quantitatively closer to contemporaneous gain-loss utility than the ordered comparison, which makes the equilibrium generally better behaved. Moreover, the separated comparison preserves the linear nature of the static comparison, while the percentile-wise ordered comparison is nonlinear. Thus, the separated comparison is more tractable in a wide class of stochastic models, where consumption utility in future periods is some function of different random variables.

3.2 Independent lotteries

As shown in Section 2, the separated comparison corresponds to the static comparison for independent prior and updated beliefs. Moreover, if the agent compares a fixed point to his beliefs, as is the case for contemporaneous gain-loss utility, then the separated comparison is the same as the ordered comparison. However, the separated comparison does not yield the same value as the ordered comparison if two non-degenerate distributions are compared, as is the case for prospective gain-loss utility. Suppose that the agent's prior beliefs about consumption are given by the distribution function G and his updated beliefs are given by F. Then, prospective gain-loss utility under the separated comparison is given by

$$\int\int \mu(u(c) - u(r))dG(r)dF(c).$$

[7] In Appendix B, I show more generally that the ordered and separated comparisons are the same if the agent updates his beliefs in a binary manner.

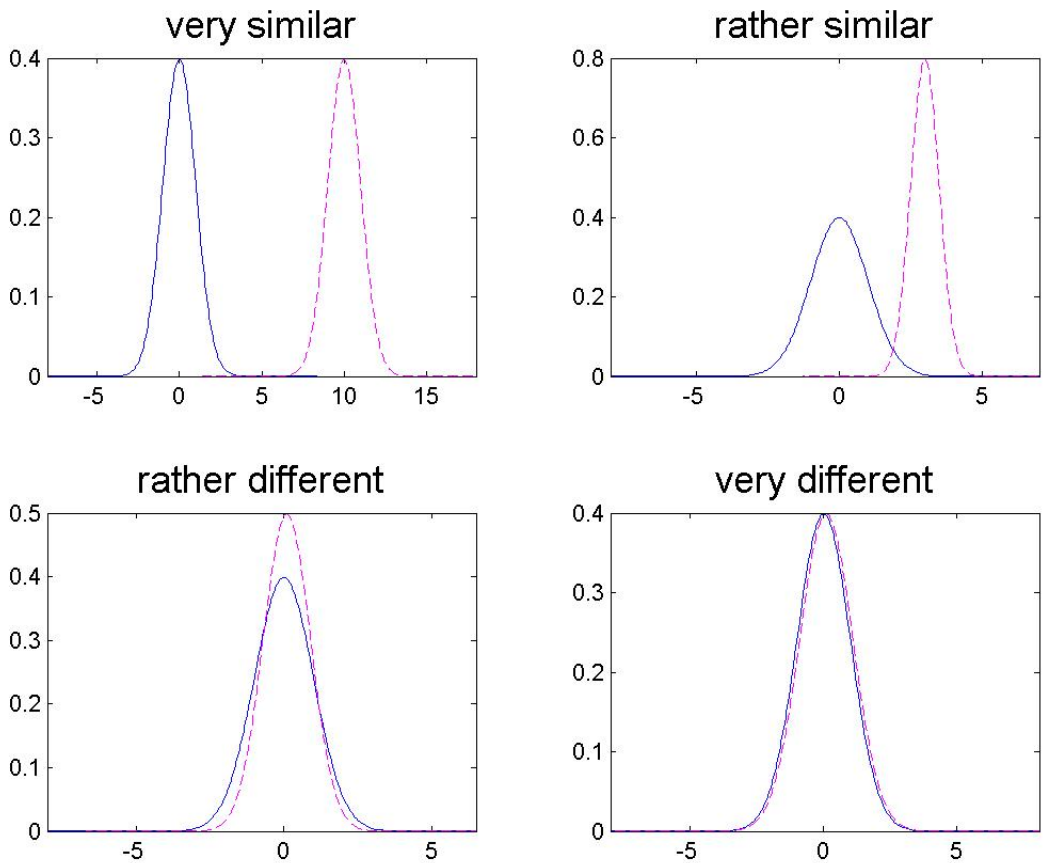

Figure 1: Similarity of the ordered and static comparisons for different prior and updated distributions.

And the ordered comparison is specified as follows

$$\int_0^1 \mu(u(C_F(p)) - u(C_G(p)))dp$$

with $C_F(p)$ the consumption level at percentile p, defined implicitly by $F(C_F(p)) \geq p$ and $F(C) < p$ for all $C < C_F(p)$. Figure 1 displays different situation in which the ordered and separated comparisons are very similar and very different for F and G being some normal distributions. It turns out that the ordered and static comparison are the same for two situations. First, if F is degenerate. Second, if the probability density functions of F and G do not overlap.

Proposition 1. *For two independent lotteries F and G, prospective gain-loss utility under the ordered and separated comparisons are equivalent if F is degenerate, i.e., $f_F(C) = 1$ for*

some C, or the probability density functions of F and G do not overlap at any point, i.e., $\{C : f_G(C) > 0 \wedge f_F(C) > 0\} = \{\}$.

This and the following propositions' proofs can be found in Appendix A. If F is degenerate, the ordered comparison and separated comparisons are the same. In this case, the agent experiences adverse gain-loss utility on average, because he is loss averse. Thus, contemporaneous gain-loss utility is negative if the mean of the distribution realizes, because the agent considers the thought of those outcomes that would have had more consumption as more painful than the thought of those outcomes that would have had less consumption as pleasurable.

Koszegi and Rabin (2009) argue in favor of the ordered comparison because the static comparison implies that the agent experiences gain-loss utility even if there is no update in information, i.e., $G = F$. As the agent is loss averse, the value of prospective gain-loss is negative under the static comparison, which seems unrealistic. Thus, among other things, the ordered comparison for prospective gain-loss utility means to capture that the agent's gain-loss utility is zero if there's no update in information. Thus, the amount of overlap of the probability density functions of F and G partly determines how different prospective gain-loss utility under the ordered and static comparison is. If the two are more distinct the two comparisons yield more similar gain-loss utility than if the two are very similar. Moreover, prospective gain-loss utility under the ordered comparison is not only zero if $F = G$ but it also features a kink at $F = G$. To understand why suppose that F and G are normal and differ by their mean only. Now if $\mu_F = \mu_G$ gain-loss utility is zero. Then, if $\mu_F > \mu_G$ the agent compares the two distributions at each percentile weighted by η as $\mu_F > \mu_G$ each percentile point is a gain. However, if $\mu_F < \mu_G$ the agent does the same comparison but now weighting each difference by $\eta\lambda$ as each percentile is a loss. Thus, in such a situation there is a kink in prospective gain-loss utility at $F = G$, which implies that there is a jump in marginal gain-loss utility under the ordered comparison. The example is illustrated in Figure 2.

Thus far, I assumed that F and G are independent. Nevertheless, I argue that in most economic models beliefs about future consumption are not independent. Independent distributions do not specify in how far the agent knew he will update his beliefs over future consumption. The agent would compare two independent distribution functions only if the update of G to F came as a surprise. In other words, F and G are independent only if G expected F with zero probability. Thus, F and G cannot be independent if the agent expected to receive information in that period and considered F as a possible updated beliefs distribution. Moreover, if G and F are not independent they are very unlikely to be the same. I.e., if the agent expected to learn in any period that future consumption is distributed according

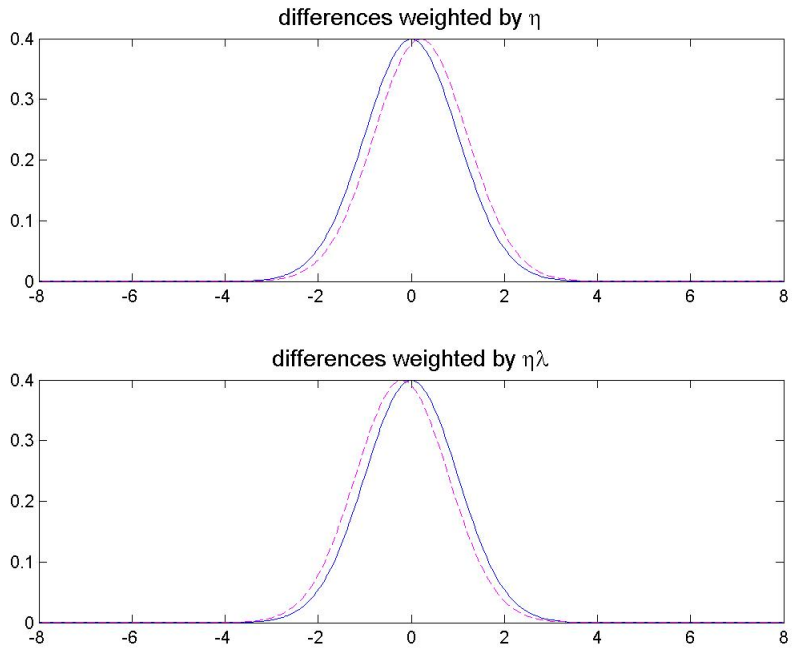

Figure 2: The ordered comparison for slightly different prior and updated distributions.

to F without certainty then his initial beliefs would correspond to $F = G$ only in knife-edge cases. Thus, the static comparison's feature, that gain-loss utility is not zero if F and G are independent and $F = G$, might not be such a disadvantage as it is not a relevant situation in many economic models, especially when there's some update in information each period. Thus, I move on to the case where updated and prior beliefs over future consumption are not independent.

3.3 Non-independent lotteries

Consider two distributions over future consumption F and G that are not independent, i.e., there exists a joint distribution function $F_{F,G}(c, r) \neq F(c)G(r)$ and a conditional distribution function $F_{G|F}(r|c) \neq G(r)$. Definition 1defines the separated comparison as follows

$$\int\int \mu(u(c) - u(r))dF_{G|F}(r|c)dF(c) = \int(\int \mu(u(c) - u(r))f_{G|F}(r|c)dr)f_F(c)dc$$

$$= \int\int \mu(u(c) - u(r))f_{F,G}(c, r)drdc.$$

Here, the agent experiences gain-loss utility only over uncertainty that has been realized in that period, as the uncertainty in F is determined by the outer integral and the inner

integral takes the state of F in its probability $f_{G|F}(r|c)$ into account. The ordered comparison is defined as above. Thus, the ordered comparison is more generally applicable as it is defined over F and G directly, whereas the separated comparison is defined over the joint distribution of F and G.

A two-period example. To illustrate the differences, I consider a specific example with two periods. Suppose consumption in the first period equals a random variable $C_1 = X$ with X being characterized by the distribution function F_X. Consumption in the second period equals the random variable $C_2 = X + Y$ with Y being characterized by the distribution function F_Y. The realizations of X and Y are denoted by x and y respectively. In the first period X realizes, with the realization denoted by $\bar{x}$, and the agent updates his beliefs about second period consumption from $X + Y$ to $\bar{x} + Y$ experiencing prospective gain-loss utility in the process. Thus, $G(x + y) = F_X(x) + F_Y(y)$ and $F(\bar{x} + y) = F_Y(y)$ with $F_{G|F}(x + y|\bar{x} + y) = F_X(x)$ and $F_{G,F}(\bar{x} + y, x + y) = F_Y(y)F_X(x)$ such that $F_{G,F} = F_{G|F}F$. Then, separated gain-loss utility according to definition 1 is given by

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mu(u(\bar{x} + y) - u(x + y))dF_X(x)dF_Y(y),$$

i.e., the separated comparison differentiates between uncertainty realized today and uncertainty realized tomorrow. In particular, the agent experiences static gain-loss utility only over that part of uncertainty that has been realized. This is different from the static comparison

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mu(u(c) - u(r))dG(r)dF(c)$$

with $G = F_X + F_Y$ and $F = x + F_Y$ which does not recognize that F and G are not independent. This non-recognition implies that the agent would experience gain-loss utility, which is negative on average as the agent is loss averse, over Y even though Y has not been realized. As above, the ordered comparison is defined as follows

$$\int_0^1 \mu(u(C_F(p)) - u(C_G(p)))dp$$

with $C_F(p)$ the consumption level at percentile p, defined implicitly by $F(C_F(p)) \geq p$ and $F(C) < p$ for all $C < C_F(p)$. In contrast to the static comparison, the ordered comparison does recognize that the two distributions are not independent because the agent experiences gain-loss utility merely over the differences in the final distribution of second-period consumption.

When a lot of information over future consumption is revealed, as the case for tail realization of X, the two comparisons approach each other as $X+Y$ and $\bar{x}+Y$ become independent and do not overlap.

Proposition 2. *For any lotteries X and Y with $F(\bar{x}+y) = F_Y(y)$ and $G(x+y) = F_X(x) + F_Y(y)$, prospective gain-loss utility under the ordered and separated comparisons are not equivalent, but approach each other for unlikely realizations of X, i.e., $X = x$ such that $f_X(x) \to 0$, and if Y approaches a degenerate lottery, i.e., $f_Y(y) = 1$ for some y.*

But, suppose that the realization of X equals it's mean $\bar{x} = E[X]$. In such a situation, contemporaneous gain-loss in period 1 will be negative, as the agent is loss averse and thus considers the comparison to the outcomes that would have had more consumption as more painful than the comparison to the outcomes that would have had less consumption. Moreover, prospective gain-loss utility under the separated comparison is quantitatively similar to contemporaneous gain-loss utility, as the separated comparison resembles the static comparison of contemporaneous gain-loss utility simply excluding remaining uncertainty. In comparison, prospective gain-loss utility under the ordered comparison is much closer to zero than under the separated comparison especially if Y has a wide distribution in comparison to X.

In such situations, where future uncertainty is very large relative to present uncertainty, there arrives information about the mean of the distribution over future consumption but very little about its variance. Thus, if $\bar{x} = E[X]$ and Y is much more variant than X the prior distribution of consumption is very close to the updated distribution over consumption. Then, the difference between the ordered and separated comparison is largest because the ordered comparison is zero whereas the separated comparison is not zero. In principle, the agent learned nothing over future consumption thus gain-loss utility should be zero. But, under the assumption of loss aversion, the agent should be more sad on average if the mean of a random variable realizes simply because he considers the thought of those outcomes that would have had more consumption as more painful than the thought of those outcomes that would have had less consumption as pleasurable. Thus, one might argue against the agent experiencing adverse contemporaneous gain-loss utility but zero prospective gain-loss utility under the ordered comparison if $\bar{x} = E[X]$.

Now, suppose that the realization of X is slightly larger than its mean $\bar{x} = E[X] - \epsilon$ with $\epsilon \to 0$ and $\epsilon > 0$. Under the ordered comparison, the agent compares a distribution which is slightly shifted to the original distribution and compares the two at each percentile. Then, each percentile represents a loss as the new distribution is slightly lower than the original one. However, if $\epsilon < 0$, each percentile represents a gain as the new distribution

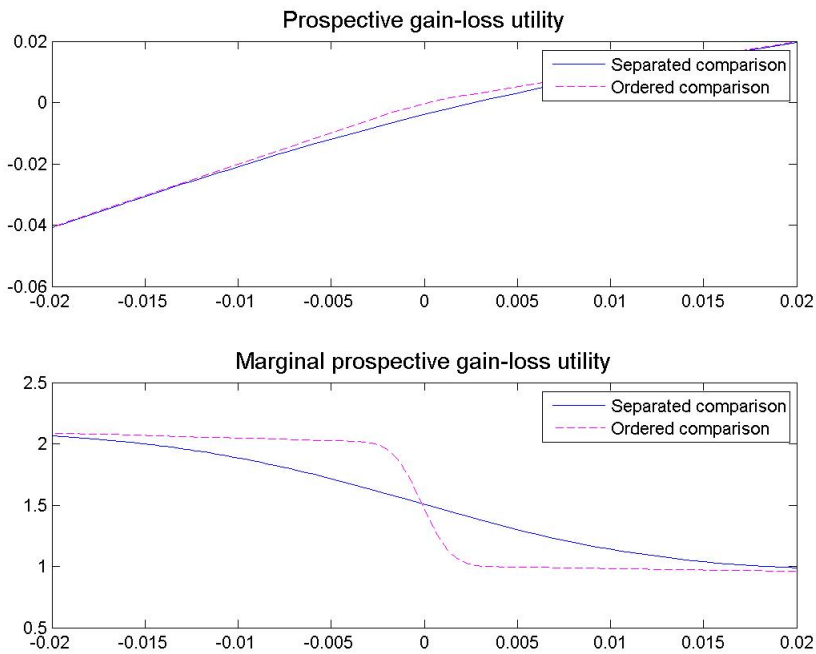

Figure 3: (Marginal) prospective gain-loss utility under the ordered and separated comparisons.

is slightly higher than the original one. Thus, there occurs a kink in prospective gain-loss utility and marginal prospective gain-loss utility jumps from η to $\eta\lambda$ in the limit when the variance of Y is very large in comparison to X. It turns out that even if the variance of Y is only somewhat larger than the variance of X prospective gain-loss utility is close to zero at $\bar{x} = E[X]$ and marginal gain-loss utility changes very quickly when moving from slightly below $E[X]$ to slightly above $E[X]$. Of course, as the variance of second period consumption changes, prospective gain-loss utility will not be exactly zero and the kink will be slightly smoothed out. Nevertheless, prospective gain-loss utility under the ordered comparison resembles a comparison to a fixed reference point whereas prospective gain-loss utility under the separated comparison resembles the comparison to a stochastic reference point and stays quantitatively similar to contemporaneous gain-loss utility in period 1 over the realization of X. Figure 3 illustrates this example for X and Y normally distributed. To illustrate the point, I chose $\sigma_Y = 4\sigma_X$, linear consumption utility $u(c) = c$, $\eta = 1$, and $\lambda = 2$. In many economic models, consumption in future periods is likely to be considerably more variable than contemporaneous consumption. As can be easily seen, even for $\sigma_Y = 4\sigma_X$, prospective gain-loss utility under the ordered comparison features a visible, only slightly smoothed out kink.

Not surprisingly, this feature of the ordered comparison brings about problems in the

agent's optimization, as the change in marginal prospective gain-loss utility is close to a jump and behaves very different from marginal contemporaneous gain-loss utility in the neighborhood of $E[X]$. Then, the first-order condition under the ordered comparison is likely to predict decreasing consumption in the realization of X, as marginal prospective gain-loss moves from η to $\eta\lambda$ very quickly. Decreasing consumption cannot be optimal as the agent would unnecessarily experience gain-loss utility and would prefer a flat consumption region, which brings about non-differentiabilities in the maximization problem. In contrast, under the separated comparison, contemporaneous marginal prospective gain-loss utility is quantitatively similar to marginal contemporaneous gain-loss utility. For example, if $u(\cdot)$ is separable in X and Y, i.e., exponential or linear in this situation, then contemporaneous marginal gain-loss utility corresponds to prospective marginal gain-loss utility under the separated comparison just weighted by first period marginal utility instead of second period marginal utility. Thus, the first-order condition under the separated comparison is very tractable and well behaved. Moreover, the separated comparison is a linear operator as opposed to the ordered comparison which yields more tractable solutions in stochastic models.

A two-period discrete lottery example. For further illustration, I outline a discrete lottery example with linear consumption utility $u(c) = c$, $\eta = 1$, and $\lambda = 2$. Consumption in period 1 is $C_1 = X = \{-1, 0, 1\}$ each with probability $\frac{1}{3}$ and consumption in period 2 is $C_2 = X + Y$ with $Y = \{0, 1\}$ each with probability $\frac{1}{2}$. Suppose $\bar{x} = E[X] = 0$ realizes. In period 1, under the separated as well as ordered comparison contemporaneous gain-loss utility is given by

$$\frac{1}{3}\eta(u(0) - u(-1)) + \frac{1}{3}\eta\lambda(u(0) - u(1)) = -\frac{1}{3}\eta(\lambda - 1) = -\frac{1}{3}.$$

In period 1, the agent's prospective gain-loss utility about period 2 consumption under the separated comparison is given by

$$\frac{1}{2}\frac{1}{3}\eta(u(0) - u(-1)) + \frac{1}{2}\frac{1}{3}\eta\lambda(u(0) - u(1)) + \frac{1}{2}\frac{1}{3}\eta(u(1) - u(0)) + \frac{1}{2}\frac{1}{3}\eta\lambda(u(1) - u(2))$$

$$= \frac{1}{6}\eta - \frac{1}{6}\eta\lambda + \frac{1}{6}\eta - \frac{1}{6}\eta\lambda = -\frac{2}{6}\eta(\lambda - 1) = -\frac{1}{3}.$$

The agent's prior beliefs about period 2 consumption are given by $\{-1, 0, 0, 1, 1, 2\}$ each with probability $\frac{1}{6}$ and the updated beliefs are given by $\{0, 1\}$ each with probability $\frac{1}{2}$. Thus, under

the ordered comparison prospective gain-loss utility is given by

$$\frac{1}{6}\eta(u(0)-u(-1))+\frac{1}{6}\eta\lambda(u(1)-u(2))=-\frac{1}{6}\eta(\lambda-1)=-\frac{1}{6}.$$

If $x=-1$ or $x=1$ realizes then contemporaneous gain-loss utility is -2 and 1 respectively and prospective gain-loss utility is-2 and 1 respectively under both the separated and ordered comparison. Thus, the lottery example illustrated the same point as the continuous outcome example above: the two comparisons yield the same answer for high and low realizations of X. But, if $\bar{x}=E[X]$ realizes the separated comparison yields a value in line with contemporaneous gain-loss utility, $-\frac{1}{3}$ in this example, while the ordered comparison yields a value closer to zero, $-\frac{1}{6}$ in this example. I chose a lottery example with $\sigma_X \approx \sigma_Y$. If $\sigma_Y > \sigma_X$ the value of prospective gain-loss under the ordered comparison would be closer to zero, and thus more different from the value of contemporaneous gain-loss utility or prospective gain-loss utility under the separated comparison.

The Lucas-tree example. An example of an economic model that follows the above example rather closely is a Lucas (1978) tree model in which the sole source of consumption is an everlasting tree that produces C_t units of consumption each period t. Consumption growth is log-normal as in Mehra and Prescott (1985), i.e., the endowment economy's exogenous consumption process is given by

$$log(\frac{C_{t+1}}{C_t})=\mu_c+\varepsilon_{t+1} \text{ with } \varepsilon_{t+1}\sim N(0,\sigma_c^2).$$

The price of the Lucas tree in each period t is P_t. The period $t+1$ return of holding the Lucas tree is then $R_{t+1}=\frac{P_{t+1}+C_{t+1}}{P_t}$. Each period t, the agent faces the price of the Lucas tree P_t and, acting as a price taker, optimally decides how much to consume C_t^*. The agent's beliefs about consumption in any future period $t+\tau$ is denoted by $F_{t+\tau}^t$. The joint distribution function of the agent's period t and period $t-1$ beliefs about consumption in any future period $t+\tau$ is denoted by $F_{t+\tau}^{t,t-1}$. Thus, the agent's maximization problem in period t is given by

$$max_{C_t}\{u(C_t)+n(C_t,F_t^{t-1})+\gamma\sum_{\tau=1}^{\infty}\beta^\tau \boldsymbol{n}(F_{t+\tau}^{t,t-1})+E_t[\sum_{\tau=1}^{\infty}\beta^\tau U_{t+\tau}]\}.$$

Here, contemporaneous gain-loss utility is given by

$$n(C_t,F_t^{t-1})=\int_{-\infty}^{\infty}\mu(u(C_t)-u(c))dF_t^{t-1}(c).$$

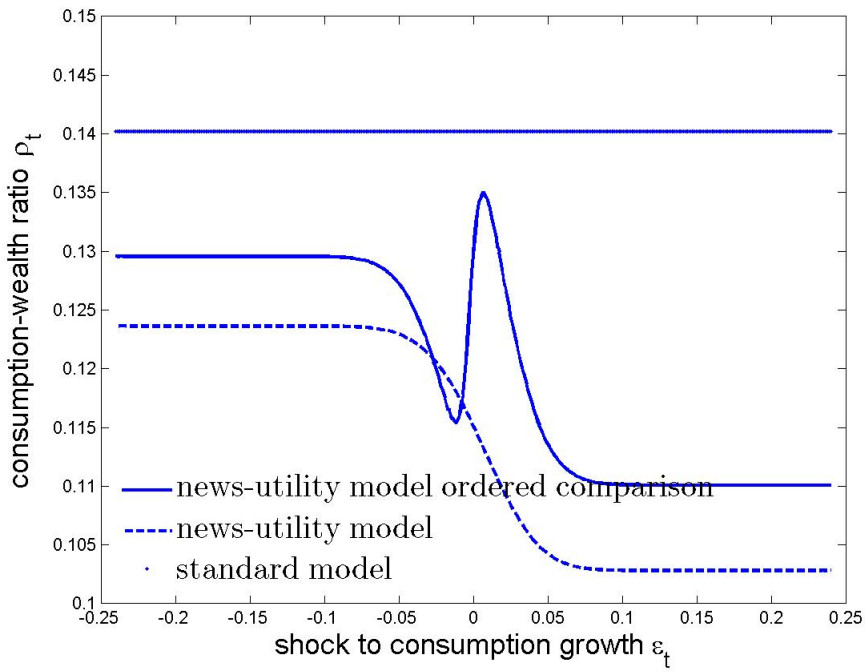

Figure 4: The consumption-wealth ratio in the Lucas-tree model under the ordered and separated comparisons.

And prospective gain-loss utility under the separated comparison is given by

$$\boldsymbol{n}(F_{t+\tau}^{t,t-1}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mu(u(c) - u(r))dF_{t+\tau}^{t,t-1}(c,r).$$

The agent's wealth in the beginning of the period W_t is determined by the budget constraint

$$W_t = (W_{t-1} - C_{t-1})R_t. \tag{1}$$

After R_t realizes, the agent optimally decides how much to consume C_t^* and how much to invest $W_t - C_t^*$. In equilibrium, the price of the tree $P_t = W_t - C_t$ adjusts so that the single agent in the model always chooses to hold the entire tree and consume the tree's entire payoff $C_t^* = C_t$ as given by the endowment economy's exogenous consumption process. In principle, this set-up corresponds to the infinite-horizon version of the above example. The model's equilibrium can be analyzed by means of the consumption-wealth ratio $\rho_t = \frac{C_t}{W_t}$ how much the agent consumes out of his wealth, as shown in Pagel (2012b).

Figure 4 displays the equilibrium consumption-wealth ratio under the ordered and separated comparisons. Under the separated comparison, the consumption-wealth ratio is decreasing in the realization of the shock to consumption growth ε_t. The agent consumes

more out of his wealth for low realizations and less for high realizations. Under the ordered comparison, the consumption-wealth ratio is decreasing most of the time. However, around the mean of the shock $\varepsilon_t = 0$, marginal prospective gain-loss utility almost jumps from $\eta\lambda$ to η such that the agent suddenly cares much less about the future, which implies that the consumption-wealth ratio is suddenly much higher, i.e., the agent consumes more out of his wealth. After the smooth jump, prospective gain-loss is similar to the separated comparison again and the consumption-wealth ratio is decreasing in the shock realization. Here, prospective gain-loss utility is the sum of infinitely many future periods. Prospective gain-loss utility for period $t+1$ consumption under the ordered comparison looks fairly similar to contemporaneous gain-loss utility because the shock ε_t contains a lot of information. However, for period $t+10$ consumption prospective gain-loss utility under the ordered comparison features a only slightly smoothed kink, which is not the case for contemporaneous gain-loss utility; nevertheless, $\gamma\beta^{10}$ is not too close to zero. Overall prospective gain-loss utility under the ordered comparison is rather kinky, which brings about the funky behavior in the agent's first-order condition and the spike of the equilibrium consumption-wealth ratio.

3.4 A simple example about responses to signals

In the following, I illustrate another simple model in which the agent receives a signal about the realization of his investment and then decides if he wants to "look up" the true realization of his investment or not. The separated comparison implies that the agent is more likely to look up his investment after having received a favorable signal. This behavior is commonly known as the "Ostrich effect" and supported by empirical evidence (Karlsson, Loewenstein, and Seppi (2009)). In contrast, the ordered comparison implies that the agent always prefers to look up his investment especially for realizations of the signal around its mean.

I quickly outline the simple model. Suppose the agent lives for three periods $t \in \{0, 1, 2\}$, in the first of which he invests his endowment of one dollar, in the second of which he receives a signal about the return of his investment, and in the third of which he consumes his entire pay off. Let the investment be sufficiently profitable such that the agent invests his entire dollar in period zero. Consumption in period two equals the investment's return $C_2 = R = e^r \sim log{-}N(\mu, \sigma^2)$, with $F_r = N(\mu, \sigma^2)$. In period one, a signal about the return R arrives given by $\tilde{R} = e^{r+\epsilon} \sim log{-}N(\mu, \sigma^2{+}\sigma_\epsilon^2)$, with $F_\epsilon = N(0, \sigma_\epsilon^2)$ and $F_{r+\epsilon} = N(\mu, \sigma^2{+}\sigma_\epsilon^2)$; after observing the signal, the agent decides if he wants to look up his investment and find out what the true realization of R is or wait until period two to find out what the true realization is. Instantaneous utility in period one is then either prospective gain-loss utility over the realization of R or prospective gain-loss utility over the signal $\tilde{R}$. The first of which

is the same for all comparisons, the latter of which is different. Under the ordered comparison prospective gain-loss utility is given by

$$\gamma\beta \int_0^1 \mu(u(C_F(p)) - u(C_G(p)))dp$$

with $C_F(p)$ the consumption level at percentile p, defined implicitly by $F(C_F(p)) \geq p$ and $F(C) < p$ for all $C < C_F(p)$ with $F = log - N(\tilde{R}, \sigma_\epsilon^2)$ and $G = log - N(\mu, \sigma^2)$. Under the separated comparison, prospective gain-loss utility is given by

$$\gamma\beta \int\int \mu(u(c) - u(r))f_{F,G}(c,r)drdc = \gamma\beta \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mu(u(e^{log(\tilde{R})-y}) - u(e^{x-y}))dF_{r+\epsilon}(x)dF_\epsilon(y),$$

because the agent separates uncertainty that has been realized in period one, represented by $\tilde{R}$ and $F_{r+\epsilon}(x)$, from uncertainty that has not been realized, represented by $F_\epsilon(y)$. Now, suppose the agent enters period one, receives the signal, decides to look up his return or not, and then experiences gain-loss utility. Then, the agent's expected gain-loss utility from looking up the return conditional on the signal $\tilde{R}$ is given by

$$\gamma\beta \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mu(u(e^{log(\tilde{R})-y}) - u(e^x))dF_r(x)dF_\epsilon(y).$$

Thus, the agent expects more favorable gain-loss utility over the return when having received a favorable signal. As can be easily seen, expected gain-loss utility from checking the return is always less than gain-loss utility from knowing merely the signal. The reason is simple, the agent expects to experience gain-loss utility, which is negative on average, over both the signal $\tilde{R}$ and the error ϵ. Thus, he always prefers to not look up his return when prospective gain-loss utility in period one is concerned. But, the difference between the two is smaller when the agent received a more favorable signal. The reason is that the expected gain-loss disutility from ϵ is less high up on the utility curve, i.e., when $\tilde{R}$ is high. Moreover, there is a benefit associated with looking up the return that is the absence of expected gain-loss disutility in period two. Thus, because the costs of looking up the return are decreasing in the signal under the separated comparison, the agent is more likely to look up his return after a favorable signal. In contrast, under the ordered comparison the agent slightly prefers to not look up his return for very good or bad realizations of the signal. For realizations of the signal around its mean, the agent prefers to find out what the return is. For illustration, Figure 5 displays the difference in prospective gain-loss if the agent does not look up the return or does. Under the separated comparison the costs are decreasing in $\tilde{R}$ in line with the Ostrich effect, whereas this is not the case under the ordered comparison.

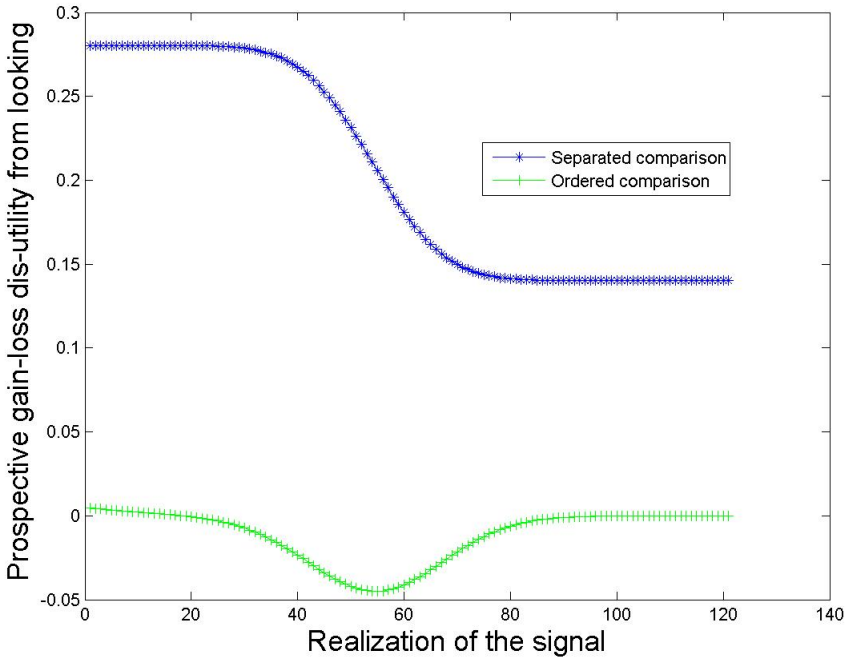

Figure 5: Prospective gain-loss utility over the realization and the signal under the ordered and separated comparisons.

4 Conclusion

Koszegi and Rabin (2009) develop a dynamic version from the static model of expectations-based reference-dependent preferences introduced by Koszegi and Rabin (2006, 2007). In the dynamic model, the agent experiences both contemporaneous gain-loss utility over present consumption and prospective gain-loss utility over changes in beliefs about future consumption. This paper proposes to calculate prospective gain-loss utility using what I call the "separated comparison" instead of the ordered comparison suggested by Koszegi and Rabin (2009). As does the ordered comparison, the separated comparison generalizes the static comparison of Koszegi and Rabin (2006, 2007). In particular, the static comparison is generalized to explicitly take potential non-independence of the fully probabilistic prior and updated beliefs about future consumption into account, which preserves the outcome-wise nature of the static comparison. Under the separated comparison the agent experiences gain-loss utility only over uncertainty that has been realized, because he considers it separately from remaining future uncertainty. Moreover, the separated comparison yields simple, tractable, and well-behaved equilibria in a large class of economic models, because prospective gain-loss utility is calculated in a linear fashion and resembles contemporaneous gain-loss utility more closely. Static expectations-based reference-dependent preferences have been widely applied

and enjoy vast popularity, whereas no attempts have been made to incorporate the dynamic preferences into economic models. I hope that this paper will contribute and facilitate more applications of the dynamic preferences in economic models.

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A Proofs

A.1 Proof of proposition 1

Proof. If $f_F(C) = 1$ for some C. The static comparison is given by

$$\int_{-\infty}^{\infty} \mu(u(C) - u(C_G))dG(C_G).$$

and the ordered comparison is given by

$$\int_0^1 \mu(u(C) - u(C_G(p)))dp$$

with $C_G(p)$ the consumption level at percentile p, defined implicitly by $G(C_G(p)) \geq p$ and $G(C) < p$ for all $C < C_G(p)$. It can be easily seen that the two are the same as C is deterministic.

If$\{C : f_G(C) > 0 \wedge f_F(C) > 0\} = \{\}$. The static comparison is given by

$$\int\int \mu(u(C_F) - u(C_G))dG(C_G)dF(C_F).$$

The agent compares each possible outcome under his updated beliefs to each possible outcome under his prior beliefs. If $\{C : f_G(C) > 0 \wedge f_F(C) > 0\} = \{\}$ and wlog F stochastically dominates G then this boils down to subtracting the expectation of G from each point of F weighted by η. Each possible point of F is weighted by it's probability so if $\{C : f_G(C) > 0 \wedge f_F(C) > 0\} = \{\}$ then the agent simply compares the expected value of F and G and multiplies the difference η (or $\eta\lambda$ if G dominates F). The ordered is given by

$$\int_0^1 \mu(u(C_F(p)) - u(C_G(p)))dp$$

with $C_F(p)$ the consumption level at percentile p, defined implicitly by $F(C_F(p)) \geq p$ and $F(C) < p$ for all $C < C_F(p)$.If $\{C : f_G(C) > 0 \wedge f_F(C) > 0\} = \{\}$ and wlog F stochastically dominates G then again this comparison boils down to subtracting the expectation of G from the expectation of F weighted by η (or $\eta\lambda$ if G dominates F). □

A.2 Proof of proposition 2

This follows directly from proposition 1.

B Ordered versus separated comparison for binary lotteries

One of the major differences between the ordered and separated comparisons appears to be the kink in prospective gain-loss utility at the expected value. Thus, if the agent updates his information about future consumption in a binary manner the two comparison could yield the same value of prospective gain-loss utility as the agent's uncertainty resolved in the current period is either a loss or a gain. This section compares the ordered and separated comparison for binary lotteries. First, suppose the agent expected a binary lottery and updates his beliefs to another binary lottery. It turns out that ordered and separated comparisons yield the same value for prospective gain-loss utility if the two lotteries are not the same which is a corollary of proposition 1.

Corollary 1. *For two independent binary lotteries F and G, prospective gain-loss utility under the ordered and separated comparisons are equivalent, if $F \neq G$.*

Proof. $F = \{x_{F1} : p, x_{F2} : 1-p\}$ and $G = \{x_{G1} : q, x_{G2} : 1-q\}$ then prospective gain-loss utility under the static comparison is given by

$$pq\mu(u(x_{F1}) - u(x_{G1})) + p(1-q)\mu(u(x_{F1}) - u(x_{G2}))$$

$$+(1-p)q\mu(u(x_{F2}) - u(x_{G1})) + (1-p)(1-q)\mu(u(x_{F2}) - u(x_{G2}))$$

and prospective gain-loss utility under the ordered comparison is given by wlog $p, q < \frac{1}{2}$ and $p < q$

$$p\mu(u(x_{F1}) - u(x_{G1})) + (q-p)\mu(u(x_{F2}) - u(x_{G1})) + (1-q)\mu(u(x_{F2}) - u(x_{G2}))$$

(one case for illustration) the first can be modified to yield the latter by adding and subtracting $p(1-q)\mu(u(x_{F1}) - u(x_{G1}))$ and $p(1-q)\mu(u(x_{F2}) - u(x_{G2}))$ and then modifying $p(1-q)\mu(u(x_{F1}) - u(x_{G1}) + u(x_{G1}) - u(x_{G2}))$ and $-p(1-q)\mu(u(x_{F2}) - u(x_{G1}) + u(x_{G1}) - u(x_{G2}))$ if $u(x_{F1}) < u(x_{G1}) < u(x_{G2})$ or $u(x_{G2}) < u(x_{G1}) < u(x_{F1})$ and $u(x_{F2}) < u(x_{G1}) < u(x_{G2})$ or $u(x_{G2}) < u(x_{G1}) < u(x_{F2})$. □

Now, consider the prior example of consumption in the first and second period, but slightly modified such that the agent updates in a binary manner.

Proposition 3. *For any lotteries X and Y with $F = x + Y$ and $G = X + Y$, prospective gain-loss utility under the ordered and separated comparisons are equivalent, if X is binary.*

Proof. Suppose $X = \{x_1 : q, x_2 : (1-q)\}$ and Y some lottery with distribution function F_Y. The separated comparison is given by

$$\int_{-\infty}^{\infty} (1-q)\mu(u(x_1+Y) - u(x_2+Y))dF_Y(Y) \text{ for } X = x_1$$

$$\int_{-\infty}^{\infty} q\mu(u(x_2+Y) - u(x_1+Y))dF_Y(Y) \text{ for } X = x_2$$

and the ordered comparison is given by

$$\int_{0}^{1} (1-q)\mu(u(x_1+Y(p)) - u((x_2+Y(p)))dp \text{ for } X = x_1$$

$$\int_{0}^{1} q\mu(u(x_2+Y(p)) - u((x_1+Y(p)))dp \text{ for } X = x_1$$

with $Y(p)$ the value of Y at percentile p, defined implicitly by $F_Y(Y(p)) \geq p$ and $F(Y) < p$ for all $Y < Y(p)$. □