

A Mechanism for the Allocation of Corporate Investment

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I. Introduction

Corporate investment in an economy without a complete set of contingent claims markets has the characteristic of a public good in the sense that the stockholders' consumption plans cannot be separated from, but depend on, the specific investment plans of the firms. Drèze [4] has shown that a constrained Pareto optimal (CPO) allocation of investment in a stock market economy must satisfy a generalization of the Samuelson [24] condition for efficient production of public goods: the investment plan should maximize a weighted sum of the stockholders' personal valuations of future output minus current input cost. However, except for those special cases in which CPO investment plans are unanimously supported by stockholders (see [17], [20], and [2]), the theory of the firm in incomplete markets lacks a suitable maximization criterion. Although the Drèze-Samuelson condition is a most appealing candidate, it is not unanimously preferred by stockholders, each of whom prefers that his or her own valuation of future output receives all the weight in the investment decision. Furthermore, the application of the Drèze-Samuelson condition depends on the correct revelation of stockholders' preferences, which, in the absence of special inducements, cannot be expected from economic agents.¹

The purpose of this paper is to develop an internal allocation mechanism capable of attaining production plans that are unanimously preferred by stockholders and that satisfy a natural notion of optimality applicable to the stock market economy. Drèze [4] has proposed the application of the Drèze-de la Vallée Poussin [5]—Malinvaud [19] mechanism (the MDP mechanism) for the allocation of public goods to solve the problem of corporate investment. The MDP mechanism obtains unanimous support for an investment plan satisfying the Drèze-Samuelson condition by sidepayments from those stockholders who bene-

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¹ Related work by Gevers [8] has shown that only under very special circumstances can majority voting be relied upon to enforce the Drèze-Samuelson condition. Allocation of public good through majority voting received a detailed treatment in the pioneering contribution of Bowen [3].

fit from the plan to those stockholders who do not. However, stockholders are assumed to adopt minimax strategies. If stockholders behave competitively, the equilibrium generated by the MDP mechanism will not generally satisfy the Drèze-Samuelson condition.² Sidepayments among stockholders are also assumed by Grossman and Hart, who do not rely on a public good allocation mechanism but on a manager who can “learn about the preferences of the firm’s stockholders” ([11], p. 301). Moreover, if the manager does not act according to those preferences, a takeover by a “new manager” is supposed to occur. Grossman and Hart recognize, however, that the takeover mechanism might not work “because of the difficulty in identifying the (stockholders’) marginal rates of substitution” (p. 302).³

Helpman and Razin [15] have proposed a solution to the problem studied in this paper when each firm has access to a single production activity, but existence of equilibrium is not assured under their scheme. In a recent paper, Forsythe and Suchanek [7] propose a satisfactory solution for the case of a single production activity. Their approach is somewhat similar to the one developed in this paper for the case in which each firm has access to many production activities.⁴

The organization of this paper is as follows: Section II considers the allocation of corporate investment as a public good using the standard two-date model of an uncertain economy. Known results on equilibrium and optimality applicable to this model are summarized in Section II.B. The notion of an allocation mechanism for corporate investment (a charter) is formalized in II.C. Section II.D. develops a mechanism under which stockholders respond to “personalized” prices by choosing their most preferred production plans. Under the assumption that stockholders behave as signal-takers and do not engage in strategic gaming, it is shown that an equilibrium relative to this mechanism exists and satisfies the applicable notion of optimality (presented in II.B.). Section III discusses some extensions and evaluates the results of this paper as an alternative to the creation of assets derived by the market.

II. Corporate Investment as a Public Good

A. The Model

We adopt Drèze’s [4] formulation of the two-date uncertain economy. There are I consumers, indexed $i = 1, \dots, I$, and J firms, indexed $j = 1, \dots, J$. There is a single physical commodity which at date 0 can be used for consumption or as input of production. The output of the firms will be available for consumption at date 1 but it is unknown at date 0. The output’s uncertainty will be

² Under certain rather strict assumptions, Henry [16] is able to show that a modified version of the discrete MDP mechanism converges to a Pareto optimum when each agent assumes the others will play the same strategy as in the previous period.

³ A form of the take-over bid mechanism was rigorously analyzed by Hart [14] with rather negative conclusions. More recently, Grossman and Hart [12] have studied how the effectiveness of take-over mechanisms is increased when stockholders exclude themselves from sharing in all the improvements produced by the raider.

⁴ The author thanks the referee for bringing to his attention the contributions of Helpman and Razin [15] and Forsythe and Suchanek [7].

resolved at date 1 according to the occurrence of one of Θ mutually exclusive states of the world, indexed $\theta = 1, \dots, \Theta$. A market for shares in firms operates at date 0.

A *consumption plan* for i is a vector $x^i = (x_0^i, x_1^i) \in R_+^{\Theta+1}$, where $x_0^i \in R_+$ is consumption at date 0 and $x_1^i = (x_{11}^i, \dots, x_{1\Theta}^i) \in R_+^{\Theta}$ is consumption at date 1.⁵ Consumer i is assumed to have a complete preference preordering \geq_i over $R_+^{\Theta+1}$ which is representable by a nondecreasing continuously differentiable function $U^i(x^i)$. $U_0^i = \partial U^i / \partial x_0^i > 0$; $U_\theta^i = \partial U^i / \partial x_{1\theta}^i = 0 \ \forall \theta$ when $x_0^i = 0$; $U_\theta^i > 0$ and U^i is strictly quasiconcave when $x_0^i > 0$. $\phi_\theta^i = U_\theta^i / U_0^i$ denotes the i th consumer's marginal rate of substitution (MRS) between $x_{1\theta}^i$ and x_0^i . $\phi^i = \{\phi_\theta^i\} \ \forall \theta$ and $\phi = \{\phi^i\} \ \forall \theta$. Furthermore, x denotes the set of consumption plans $\{x^i\} \ \forall i$.

A *production plan* for firm j is a vector $y^j = (y_0^j, y_1^j) \in Y^j \subset R_+^{\Theta+1}$, where $y_0^j \in R_+$ is input at date 0 and $y_1^j = (y_{11}^j, \dots, y_{1\Theta}^j) \in R_+^{\Theta}$ is output at date 1. The production set Y^j is defined by a continuously differentiable convex function $f^j(y^j) \leq 0$, with $f_0^j = \partial f^j / \partial y_0^j < 0$, $f_\theta^j = \partial f^j / \partial y_{1\theta}^j > 0$, and $f^j(0) = 0$.⁶ $\psi_\theta^j = -f_\theta^j / f_0^j$ denotes the marginal cost of $y_{1\theta}^j$. $\psi^j = \{\psi_\theta^j\} \ \forall \theta$. Furthermore, y denotes the set of production plans $\{y^j\} \ \forall j$.

The initial endowment of consumer i consists of an amount $w_i^0 > 0$ of the physical commodity and shareholdings $\omega_i^0 = (\omega_{i1}^0, \dots, \omega_{iJ}^0) \in R_+^J$ in the J firms. $\sum \omega_{ij}^0 = 1 \ \forall j$. w denotes $\{w_i^0\} \ \forall i$. Final shareholdings are $\omega_i \in R_+^J$, $\sum \omega_{ij} = 1 \ \forall j$. $\Omega^0 = [\omega_{ij}^0]$ and $\Omega = [\omega_{ij}]$ are the $I \times J$ matrices of initialⁱ and final shareholdings, respectively. p_j is the market price of firm j , $p = \{p_j\} \ \forall j \in R_+^J$.

In this model, a *stock-market feasible allocation* is an array (x, y, Ω) such that

$$\begin{aligned} \sum_i x_0^i + \sum_j y_0^j &\leq \sum_i w_i^0 \\ x_1^i &\leq \sum_j \omega_{ij} y_1^j \quad \forall i \\ f^j(y^j) &\leq 0 \quad \forall j. \end{aligned}$$

Furthermore, a *constrained Pareto optimum* (CPO) is a stock-market feasible allocation (x, y, Ω) such that there exists no stock-market feasible allocation $(\hat{x}, \hat{y}, \hat{\Omega})$ with $\hat{x}^i \geq x^i \ \forall i$ and $\hat{x}^i > x^i$ for some i .

⁵ R_+^N denotes the nonnegative orthant of the N -dimensional Euclidean space, R^N .

⁶ Note that Y^j is allowed to be strictly convex. Such a case is compatible with a limited number of firms only in the presence of set-up costs, which though not explicitly introduced into the model, are assumed to have determined the number of existing firms. As Grossman and Hart [11] have pointed out, a limited number of firms is essential to the analysis of stockholder conflict in incomplete markets, for otherwise each consumer would end up owning his own firm and conflict would disappear.

B. Equilibrium and Optimality

The following natural equilibrium concept for the stock market economy was introduced by Drèze [4]:

Definition 1. A *Drèze equilibrium*, relative to the initial endowment (w^0, Ω^0) is an array (x, y, Ω, p, ϕ) such that

- (i) (x, Ω, p) is an *exchange equilibrium* relative to the production plans y . That is

$$\left(\hat{x}_0^i, \sum_j \hat{\omega}_{ij} y_1^j \right) > x^i$$

implies

$$\hat{x}_0^i + \sum_j \hat{\omega}_{ij} (p_j + y_0^j) > x_0^i + \sum_j \omega_{ij} (p_j + y_0^j) \quad \forall i$$

and

$$x_0^i + \sum_j \omega_{ij} (p_j + y_0^j) \leq w_i^0 + \sum_j \omega_{ij}^0 p_j \quad \forall i$$

$$x_1^i \leq \sum_j \omega_{ij} y_1^j \quad \forall i$$

$$\omega_{ij} \geq 0 \quad \forall i, j$$

$$\sum_i \omega_{ij} = 1 \quad \forall j.$$

- (ii) For every firm j , (x, y^j, ϕ) is a *Lindahl equilibrium* relative to shareholdings Ω . That is,

$$\hat{x}^i > x^i$$

implies

$$\hat{x}_0^i + \phi^i \hat{x}^i > x_0^i + \phi^i x_1^i,$$

and y^j maximizes

$$\sum_{\theta} y_{1\theta}^j \left(\sum_i \omega_{ij} \phi_{\theta}^i \right) - y_0^j$$

on Y^j .

Under the stated assumptions, it can be shown that a Drèze equilibrium exists (see [4], Theorem 5.3). On the other hand, it is known that there are Drèze equilibria which are not CPO [4], [14]. Moving out of these equilibria to a CPO would require simultaneous adjustments in production plans and portfolios which do not seem likely in a decentralized economy. As Grossman [10] has pointed

out, in order to characterize the optimality of equilibrium in incomplete markets, a concept weaker than CPO is needed. Accordingly, let us introduce the following:⁷

Definition 2. A *Grossman optimum* is a stock-market feasible allocation (x, y, Ω) such that

- (i) There exists no other stock-market feasible allocation $(\hat{x}, y, \hat{\Omega})$ with $\hat{x}^i \geq x^i \forall i$ and $\hat{x}^i > x^i$ for some i .
- (ii) There exists no other stock-market feasible allocation $(\hat{x}, \hat{y}, \Omega)$ with $\hat{x}^i \geq x^i \forall i$ and $\hat{x}^i > x^i$ for some i .

Hence, we can state the following weak version of the fundamental theorems of welfare economics:

Theorem 1.

- (i) Every Drèze equilibrium is a Grossman optimum; and
- (ii) Every Grossman optimum such that $x_0^i > 0 \forall i$ is a Drèze equilibrium.

Theorem 1 follows from well-known results: The correspondence between Definition 1(i) and Definition 2(i) is a corollary of the correspondence between equilibrium and optimum in a competitive economy ([4], Proposition 3.3). The correspondence between Definition 1(ii) and Definition 2(ii) is a corollary of the correspondence between equilibrium and optimum in an economy with a private good x_0 and Θ public goods x_i ([4], Theorem 3.2).

Theorem 1 will be used in the sequel to characterize equilibrium relative to an investment allocation mechanism.

C. Allocation Mechanisms

Since the price mechanism of the stock market can be relied upon to produce an exchange equilibrium relative to the production plans, the question that has to be examined is the attainability of a Lindahl production equilibrium.

We assume that the solution to the investment allocation problem of the corporation is defined in its charter C which specifies: (1) the language used by stockholders to communicate their preferences to the firm, (2) the allocation rule specifying how much to produce at each state, and (3) the compensation rules specifying how stockholders are compensated for accepting the outcome of the allocation rule. The language specifies the contents of each stockholder's message as a T-vector of signals $m_j^i \in M \subset R^T$; the allocation rule is a quasiconcave function $y: M^I \rightarrow Y$; and the compensation rules are IJ quasiconcave functions $c_j^i: M^I \rightarrow R$ such that $\sum_i c_j^i = 0 \forall j$.

Although ours is a static equilibrium analysis, the following idealized description of the working of an allocation mechanism in real time will motivate the

⁷ Grossman [10] developed the notion of optimality under incomplete coordination for a two-date exchange model with many commodities. Subsequently, this notion was generalized to a many-date model with production by Grossman and Hart [11] who, in defining production optimality, use the matrix of initial shareholdings as a means of resolving the intergenerational conflict inherent to the many-date model.

assumptions made below. After the firms announce their production plans, the stock exchange meets and shares are traded until equilibrium is reached. Subsequently, the new stockholders signal their preferences about production plans to each firm. When a "signal equilibrium" is attained (i.e., no further revisions are made), the transfers among stockholders (if any) specified by the compensation rules of the charter are made and the firm revises its production plan according to the allocation rule. The stock exchange meets again to trade on the basis of the revised production plans and so on. The production plans become definitive when no stock transaction takes place following a plan revision.

Stockholder behavior under a corporate charter C is specified as follows:

Problem I (exchange optimum). At meeting t of the stock exchange, stockholder i solves

$$\max_{(x^i, \omega_i^t)} U^i(x^i)$$

subject to

$$\begin{aligned} x_0^i + \sum_j \omega_{ij}^t (p_j^t + y_0^j) &\leq w_i^t + \sum_j \omega_{ij}^{t-1} p_j^t \\ x_1^i &\leq \sum_j \omega_{ij}^t y_1^j, \quad x^i \in R_+^{\Theta+1}, \quad \omega_i^t \in R_+^J, \end{aligned}$$

for given p^t, y^j and w_i^t , where

$$w_i^t = w_i^{t-1} + \sum_j (\omega_{ij}^{t-2} - \omega_{ij}^{t-1}) p_j^{t-1} + \sum_j c_j^i,$$

and $\omega_i^{-1} = \omega_i^0 \forall i$.

Problem II (production optimum). At stockholders' meeting t of firm k , stockholder i solves

$$\max_{(x^i, m_k^i)} U^i(x^i)$$

subject to

$$\begin{aligned} x_0^i + \sum_j \omega_{ij}^{t-1} y_0^j - \sum_j c_j^i &\leq w_i^{t-1} + \sum_j (\omega_{ij}^{t-2} - \omega_{ij}^{t-1}) p_j^{t-1} \\ x_1^i &\leq \sum_j \omega_{ij}^{t-1} y_1^j \end{aligned}$$

$$x^i \in R_+^{\Theta+1},$$

$$y^k = y^k(m_k^i; m_k^l = \bar{m}_k^l \forall l \neq i),$$

$$c_k^i = c_k^i(m_k^i; m_k^l = \bar{m}_k^l \forall l \neq i),$$

where $\bar{m}_k^l \forall l \neq i$ and $y^j, c_j^{i,t} \forall j \neq k$ are given.

The solution to the convex programming problems I and II does not guarantee a global maximum of the nonconvex programming problem: $\max U^i(x^i)$ with respect to x^i , ω_i , and $m^i = \{m_j^i\} \forall j$ simultaneously.⁸ Therefore, the resulting equilibrium satisfies Grossman optimality rather than CPO. The concept of an equilibrium relative to a corporate charter is now introduced. Let us denote $m_j = \{m_j^i\} \forall i$ and $m = \{m_j^i\} \forall i, j$.

Definition 3. A Drèze equilibrium relative to a corporate charter C and the initial endowment (ω^0, Ω^0) , is an array $(x, y, \Omega, p, \hat{m})$ such that

- (i) (x, Ω, p) is an exchange equilibrium relative to the production plans y .
- (ii) The stockholders' messages \hat{m} are such that

$$(a) \quad y^j = y^j(\hat{m}_j) \quad \forall j,$$

$$(b) \quad \text{for all } i, \quad (\bar{x}^i, \bar{y}) > (x^i, y), \quad \bar{y}^j(\bar{m}^i; m_j^l = \hat{m}_j^l \quad \forall l \neq i), \text{ implies}$$

$$\begin{aligned} \bar{x}_0^i + \sum_{k \neq j} \omega_{ik} y_0^k - \sum_{k \neq j} c_k^i + \omega_{ij} \bar{y}_0^i \\ - c_j^i(\bar{m}_j^i; m_j^l = \hat{m}_j^l \quad \forall l \neq i) > x_0^i + \sum_j \omega_{ij} y_0^j - \sum_j c_j^i. \end{aligned}$$

Although a number of attractive allocation mechanisms have been developed in recent years for dealing with the public good problem, most of them cannot be adapted to produce a satisfactory specification of the corporate charter. In this paper, we are interested in mechanisms that produce individually rational and Pareto optimal allocations when each stockholder behaves competitively with respect to the messages of the other stockholders. These requirements are not simultaneously satisfied by some of the most interesting mechanisms available. The MDP mechanism utilized by Drèze [4] fails to generate Pareto optimal allocations when stockholders behave competitively. The mechanisms developed by Groves and Ledyard [13] and Green and Laffont [9] have very strong incentive properties but can force some stockholders to accept an unfavorable plan. Although whether stockholders would demand individual rationality is an open question [9], Ch. 6), its role in our model is to preclude cycling between investment and stock market allocations and thus assure the existence of a simultaneous production-exchange equilibrium.⁹

The classical Lindahl mechanism in which stockholders communicate their MRS to the manager will not, in general, produce a Pareto optimal allocation of

⁸ The source of nonconvexity is the bilinear nature of the date-1 constraint. See Drèze [4].

⁹ At some expense in terms of the realism of the model, the notion of equilibrium and optimality adopted in this paper can be redefined to make feasible the application of non-individually rational mechanisms. One needs to formulate a model which does not require iteration between investment and stock allocations and is, therefore, not subject to cycling. For example, in the Grossman-Hart [11] model, iteration between investment and stock allocations is ruled out. This is done by defining optimality from the viewpoint of initial stockholders and assuming that production plans cannot be revised by final stockholders.

investment. It is instructive to analyze the nature of this failure. The Lindahl Charter is defined as follows:

The message from stockholder i is $m_j^i = \hat{\phi}_j^i$, where $\hat{\phi}_j^i$ is the vector of the MRS between x_{10}^i and x_0^i revealed to firm j . The allocation rule is $y^j \in Y^j$ which maximizes

$$\sum_0 y_{10}^j \left(\sum_i \omega_{ij} \hat{\phi}_{j0}^i \right) - y_0^j$$

and there is no compensation, i.e., $c_j^i \equiv 0 \forall i, j$.

As in the standard public good problem, stockholders cannot be expected to truthfully report their MRS. In fact, each stockholder will choose $(x^i, \hat{\phi}_j^i)$ as the solution to Problem II above, an interior solution of which requires that $\hat{\phi}_j^i$ be reported such that, given the allocation rule $y^j = y^j(\phi_j^i; \hat{\phi}_j^l = \bar{\phi}_j^l \forall l \neq i)$, the stockholders true MRS vector equates marginal costs: $\phi^i = \Psi^j$. Note that this is not a “free-rider” problem. In the classic public good allocation problem, the consumer finances a fraction of the cost of the good but consumes all of it. Hence, the consumer attempts to free-ride by understating his or her preferences. In the corporate investment problem, on the other hand, the stockholder cannot free-ride because his or her share of output is limited to his or her ownership fraction. Nevertheless, the stockholder misstates his or her preferences as a way to obtain the investment plan most suited to his or her own tastes: he or she prefers $\phi^i = \Psi^j$ while an interior Lindahl equilibrium (Definition 1(ii)) requires the satisfaction of the Drèze-Samuelson condition

$$(1) \quad \sum_i \omega_{ij} \phi^i = \psi^j.$$

The problem with the Lindahl Charter is that it generally will fail to produce an equilibrium. Existence of the latter would require that investment plans satisfying $\sum_i \omega_{ij} \phi_j^i = \Psi^j$ would also satisfy $\phi^i = \Psi^j \forall i$. It is well known, however, that this last equality is generally unattainable in an incomplete market. Equality of the relevant MRS across individuals is attainable under suitable restriction in preferences and beliefs (see [23] and [20], Ch. 8 or in technology see [6] and [2]).¹⁰ Such cases however, are not interesting in the present context because in them stockholders unanimity is assured and the firm can recover the true MRS from stock prices. The following section develops a corporate charter for the more general case in which unanimity is not assured *a priori*.

D. A Solution to the Corporate Investment Problem

The standard Lindahl Charter invites misrepresentation by asking stockholders to reveal their MRS. We now follow the alternative approach of requiring each stockholder to choose his most preferred production plan. A sidepayment scheme is used to guide stockholders’ choice toward a Lindahl allocation under the assumption that stockholders behave competitively and do not engage

¹⁰ Although under Ekern-Wilson, spanning the market will not equate all MRS across individuals, it will equate the MRS pertaining to the attainable bundles of state-contingent output.

in strategic gaming. Smith's [26], [27] experimental work in public good decision mechanisms suggests that gaming is not a problem in practice.

Under the present mechanism, to be called the Signal-Taking Stockholder (STS) Charter, the stockholders of firm j propose changes in a reference production plan, \bar{y}^j , which was arrived at prior to the last meeting of the exchange (the first reference plan is a primitive component of the model). Plan \bar{y}^j can be any plan in Y^j , including $\bar{y}^j = 0$. The message from stockholder i is $m_j^i = \delta_j^i$, where $\delta_j^i \in R^0$ is a vector of increments the stockholder would like to add to the output required by the others. The allocation rule is

$$y_1^j = \bar{y}_1^j + \sum_i \delta_j^i, \quad f^j(y_0^j, y_1^j) = 0.$$

The compensation rule is based upon vectors $t_j^i \in R^0 \forall i$ of transfers per unit of state-contingent output. That is,

$$c_j^i = -t_j^i \sum_l \delta_j^l.$$

The transfer vectors are chosen by means to be discussed below such that

$$\sum_i t_j^i = 0.$$

This implies

$$\sum_i c_j^i = 0.$$

The production plan \bar{y}^j is changed to y^j only if each stockholder, knowing the transfer vector assigned to himself or herself and the other stockholders' messages, does not change his or her own. At that point every stockholder agrees on the new production plan y^j . Note that the STS Charter is individually rational. In fact, no stockholder can be forced to accept a change unfavorable to himself or herself since he or she can always choose

$$\delta_j^i = - \sum_{l \neq i} \delta_j^l$$

which gives $y^j = \bar{y}^j$ and $c_j^i = 0$.

The stockholder will choose (x^i, δ_j^i) as the solution to Problem II of Section II.C, an interior solution of which requires the satisfaction of

$$(2) \quad \omega_{ij}(\phi^i - \psi^j) = t_j^i.$$

Summing (2) over i yields (1), the Drèze-Samuelson condition. In fact, we prove in the Appendix the following

Lemma 1. There is a one-to-one correspondence between the Lindahl equilibrium of Definition 1 and the production equilibrium relative to the STS charter.

Thus, trading in the stock exchange and investment allocation according to the STS Charter both maintain individual rationality and Pareto optimality. Hence, Theorem 5.3 of Drèze [4] establishing the existence of equilibrium and Theorem 1 above on optimality apply and we can state

Theorem 2.

- (i) There exists a Drèze equilibrium relative to the STS Charter.
- (ii) Every Drèze equilibrium relative to the STS Charter is a Grossman optimum and every Grossman optimum such that $x^i > 0 \forall i$ is a Drèze equilibrium.

The following are possible alternative means of generating the transfer vectors of the STS Charter. (a) Management can act as a Walrasian auctioneer (and adjust the transfer vector as in Arrow and Hahn ([1], pp. 277-288), for example).¹¹ (b) As in Smith's [27] auction mechanism, each stockholder can bid his own t_j^i . His message then becomes $m_j^i = (\delta_j^i, t_j^i)$ and he is compensated at the rate

$$\sum_{l \neq i} t_j^l.$$

He signals agreement by communicating

$$t_j^i = - \sum_{l \neq i} t_j^l.$$

Otherwise, the proposed allocation is vetoed and $y^j = \bar{y}^j$ until a new compensation and allocation scheme is approved. (c) As in Walker's [29] auctioneerless mechanism, the transfer vector can be made a function of the others' requests; that is¹²

$$t_j^i = \delta_j^{i+2} - \delta_j^{i+1}, \quad i = 1, \dots, n, n+1 = 1, n+2 = 2.$$

The success of each of these procedures in producing a Lindahl equilibrium depends on the stockholders behaving myopically and ignoring their ability to affect their transfer vectors through strategic gaming. Whether this will in fact happen under (a) is not sure. On the other hand, while the auctioneerless mechanisms (b) and (c) seem less subject to manipulation, they might be less expedient than (a), and (c) may become unstable in a large economy [21]. Determining which procedure is best in terms of nonmanipulability, speed of convergence, and stability is a subject beyond the scope of this paper which can be best studied with the tools of experimental economics [26], [27].

¹¹ This formulation of the STS Charter is similar to Malinvaud's [18] "price indicators" mechanism. Note, however, that the present does not maintain monotonic improvement during the *tâtonnement* while Malinvaud's mechanism uses lump-sum transfers for that purpose.

¹² Walker proves a one-to-one correspondence exists between the Lindahl equilibrium prices (here $t_j^i \forall i$) and the individuals' messages (here $\delta_j^i \forall i$).

III. Concluding Remarks

The theory of competitive equilibrium in a stock market economy has been significantly advanced by the remarkable contributions of Drèze [4] and Grossman and Hart [11]. Each of these papers proposes a criterion for choosing among investment plans, but does not develop a satisfactory mechanism for its implementation. In this paper, we attempted to contribute to the development of such a mechanism. A simple mechanism for attaining Lindahl production allocations was developed and shown to be suitable for its task if stockholders do not engage in strategic gaming. While gaming cannot be ruled out *a priori*, the experimental results reported by Smith [26], [27] using somewhat similar mechanisms suggest that it is not a problem.

The allocation mechanism developed in this paper can be incorporated into the many period-many good model of a competitive economy developed by Grossman and Hart [11]. For example, the STS Charter can be used by the Grossman-Hart manager to calculate sidepayments and the unanimously preferred production plan.

The allocation mechanism developed in this paper does not require observability of the states of nature. Income sharing is a function of the proceeds of the production plan and not of the realization of specific states of nature. Hence, the mechanism is not subject to the problem of costly state verification studied by Townsend [28] or to the problem of state unobservability pointed out by Satterthwaite [25], and it genuinely expands the allocation possibilities of the economy beyond those naturally developed by the market. Satterthwaite [25] has shown that there is an incentive to create derived assets which span observable states. For example, options written on return portfolios can span those states that generate distinct returns [22]. Whether the economy will develop its allocative capability through option and other contingent claims markets or through an internal allocation mechanism depends on the transaction costs associated with each alternative. However, both alternatives are complementary. Contingent claims markets are likely to be superior when state observability is not a problem. On the other hand, internal allocation mechanisms that do not rely on state verification permit resolving stockholders' conflict when states cannot be observed.

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Appendix

Proof of Lemma 1. (a) Let $(x^i, \delta_j^i, t_j^i) \forall i$ be a production equilibrium relative to the STS Charter. Then the following conditions hold for each i :

$$(A.1) \quad (\delta_j^i - \gamma_j^i) [\omega_{ij}(\phi^i - \psi^j) - t_j^i] = 0$$

where

$$\gamma_j^i = - \left(\bar{y}_1^j + \sum_{l \neq i} \delta_j^l \right) \left(\text{since } y_1^j = \bar{y}_1^j + \sum_l \delta_j^l \geq 0 \right)$$

$$(A.2) \quad \delta_j^i - \gamma_j^i \geq 0$$

$$(A.3) \quad \omega_{ij}(\phi^i - \gamma^j) - t_j^i \leq 0$$

and

$$(x_0^i, y) > (x_0^i, y),$$

where

$$\hat{y}^k = y^k \quad \forall k \neq j,$$

implies

$$(A.4) \quad \hat{x}_0^i + \omega_{ij} \hat{y}_0^j + \sum_{\theta} t_{j\theta}^i \hat{y}_{1\theta}^j > x_0^i + \omega_{ij} y_0^j + \sum_{\theta} t_{j\theta}^i y_{1\theta}^j.$$

Note that

$$y_{1\theta}^j = 0 \iff \delta_{j\theta}^i = \gamma_{j\theta}^i \quad \forall i$$

$$y_{1\theta}^j > 0 \iff \delta_{j\theta}^i > \gamma_{j\theta}^i \quad \forall i$$

since at equilibrium $\delta_{j\theta}^i = \gamma_{j\theta}^i$ holds if and only if it holds for all i . Hence, there is a one-to-one correspondence between (A.1)-(A.3) and

$$(A.5) \quad y_1^j \left(\sum_i \omega_{ij} \phi^i - \psi^j \right) = 0$$

$$(A.6) \quad y_1^j \geq 0$$

$$(A.7) \quad \sum_i \omega_{ij} \phi^i - \psi^j \leq 0,$$

which are the conditions for maximum

$$\sum_{\theta} y_{1\theta}^j \left(\sum_i \omega_{ij} \phi_{\theta}^i \right) - y_0^j$$

with respect to $y^j \in Y^j$.

(b) Since the stockholder's Problem II is convex, local satisfaction of (A.4) is a sufficient condition. Let

$$dx_0^i = \hat{x}_0^i - x_0^i, \quad dy_1^j = \hat{y}_1^j - y_1^j, \quad \text{and} \quad dx_1^i = \omega_{ij} dy_1^j.$$

Then $(dx_0^i, dy_0^j) > (0,0)$ implies

$$(A.8) \quad dx_0^i + \omega_{ij} dy_0^j + \sum_{\theta} t_{j\theta}^i dy_{1\theta}^j > 0.$$

But

$$dy_0^j - \sum_{\theta} \psi_{\theta}^j dy_{1\theta}^j = 0 \quad \text{on} \quad f^j(y^j) = 0 \quad (\text{where } \psi_{\theta}^j = -f_{\theta}^j/f_0^j).$$

Hence, (A.8) can be written as

$$(A.9) \quad dx_0^i + \sum_{\theta} (t_{j\theta}^i + \omega_{ij} \psi_{\theta}^j) dy_{1\theta}^j > 0.$$

Since, for θ such that (A.3) holds as strict inequality, $dy_{1\theta}^j = 0$ (any change involving $dy_{1\theta}^j > 0$ is clearly dominated), (A.9) implies

$$dx_0^i + \sum_{\theta} \phi_{j\theta}^i dx_{1\theta}^i = dx_0^i + \sum_{\theta} \omega_{ij} \phi_{j\theta}^i dy_{1\theta}^j > 0$$

and vice versa. Hence, there is a one-to-one correspondence between (A.4) and

$$“\hat{x}^i > x^i \quad \text{implies} \quad \hat{x}_0^i + \phi^i \hat{x}_1^i > x_0 + \phi^i x_1”.$$