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Combining Related and Sparse Data in Linear Regression Models

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Meta-analysis has become a popular approach for studying systematic variation in parameter estimates across studies. This article discusses the use of meta-analysis results as prior information in a new study. Although hierarchical prior distributions in a traditional Bayesian framework are characterized by complete exchangeability, meta-analysis priors explicitly incorporate heterogeneity in prior vectors. This article discusses the nature of the meta-analysis priors, their properties, and how they can be integrated into a familiar recursive estimation framework to enhance the efficiency of parameter estimates in linear regression models. This approach has the added advantage that it can provide such estimates when (a) the design or data matrix is not of full rank or (b) when observations are too few to allow independent estimation. The methodology is illustrated using published and new meta-analysis results in market-response and diffusion-of-innovation models.

KEY WORDS: Exchangeability; Meta-analysis; Prior information; Random coefficient regression; Recursive estimation.

1. INTRODUCTION

Meta-analysis has been promoted in a variety of fields as a way to integrate research findings across various studies (Farley and Lehmann 1986; Glass, McGaw, and Smith 1981; Hite and Fraser 1988; Hunter, Schmidt, and Jackson 1982; Peterson, Albaum, and Beltramini 1985; Sultan, Farley, and Lehmann, in press). Within an analysis of variance (ANOVA) framework, the various studies are viewed as imperfect replications of one overall but unplanned experiment. This replication framework generalizes and enhances the understanding of systematic variations in research findings based on design characteristics. Some applications have appeared in the literature in recent years and a number of interesting generalizations have been established and discussed (e.g., see Assmus, Farley, and Lehmann 1984; Farley, Lehmann, and Ryan 1981, 1982; Houston, Peter, and Sawyer 1983; Reilly and Conover 1983).

Meta-analysis findings, however, have rarely been incorporated as a source of prior knowledge or information in subsequent studies. Given the design characteristics of a new study, it should be rather simple to derive some prior parameter estimates using meta-analysis results. When prior knowledge is updated with sample information on the new situation, resulting parameter estimates tend to be more stable and efficient (e.g., see Lilien, Rao, and Kalish 1981); hence an attempt should be made to include prior information in the estimation procedure.

Gain in efficiency from the use of prior information has been the reason for extensive use and development of Bayesian statistical methods. The work by Lindley and Smith (1972) advocating the use of hierarchical prior distributions has become the foundation for numerous applications. Early research on integrating independently generated research findings into priors adopted this approach (DuMouchel and Harris 1983; Kuczera 1983). The strong prior assumption of exchangeability underlying hierarchical prior distributions has been criticized, however, as an oversimplification (Kass 1983). The assumption, usually captured in a normal prior (Leamer 1978, p. 49), states that the research findings are related and that the data carry information about that relationship. Because of differences in the research studies, heterogeneity is almost certain to enter that relationship. Since prior information on heterogeneity is often sparse, neither grouping nor the use of conditional priors has been practical, and the traditional approach, with its strict assumption of equal information in all of the prior studies, has survived valid criticisms.

In attempting to uncover systematic variations, meta-analysis results explicitly recognize parameter heterogeneity across independently executed studies. Hence using the results as prior information circumvents the strict exchangeability assumption underlying hierarchical Bayes priors and addresses the valid concern often raised in empirical research relying on such priors. The power of the meta-analysis model in this research is that it explicitly characterizes the sources of parameter heterogeneity in prior studies. Matching a new study with prior studies on the moderator variables specified in the meta-analysis model differentiates the information content of the priors. In other words, heterogeneity is explicitly recognized in prior information (Vanhonacker, in press).

Using a known recursive estimator in a linear regression framework, this article discusses how these priors can be integrated with sample information. Besides the established gain in parameter efficiency, the approach has the added advantage that it can provide updated parameter estimates in instances in which sample information is missing or insufficient to allow for independent estimation. The reported empirical illustrations are very encouraging in that respect.

2. DERIVATION OF META-ANALYSIS PRIORS IN A MULTIPLE REGRESSION FRAMEWORK

Published meta-analyses focus on individual parameters analyzed independently using ANOVA. Because the parameters in each of these studies are not estimated independently, this approach to recovering systematic variance in them has ignored their covariance structures. Casting meta-analysis within a multivariate regression framework allows the incorporation of those covariances that, in a familiar generalized least squares (GLS) framework, enhance the efficiency of the results and sharpen the statistical power of the significance tests. Although conceptually not different from an ANOVA approach, the regression approach is adopted here because efficient priors are naturally preferred over inefficient ones. Furthermore, this approach simplifies comparisons to the traditional hierarchical Bayes method, which it improves upon by assuming partial exchangeability. To set the tone for subsequent discussion and pinpoint inherent limitations of the suggested approach, the meta-analysis priors are derived here within a multiple-regression framework.

Meta-analysis attempts to capture and explain systematic variations in parameter estimates across various studies. Hence a set of original studies is available for which linear regression models can be specified as

$$y_i = X_i\beta_i + u_i \quad \text{for } i = 1, 2, \ldots, m, \tag{1}$$

where y_i is an $(n \times 1)$ vector containing observations on the dependent variable in study i, X_i is an $(n \times p)$ matrix containing observations on p nonstochastic predictors in study i, β_i is a $(p \times 1)$ vector containing the parameters, u_i is an $(n \times 1)$ disturbance vector, and m denotes the number of previous studies. With respect to the disturbance terms, the assumptions are made that the expected error is 0 and that errors are independent both within and across studies, or

$$\begin{aligned} E(u_i) &= 0 \quad &&\text{for all } i \\ E(u_iu_j') &= \sigma_i^2 I \quad &&\text{for } i = j \\ &= 0 \quad &&\text{otherwise.} \end{aligned} \tag{2}$$

In the current development of the methodology described hereafter, the assumption of linearity in (1) is fundamental and represents a limitation. (Linearity is an implicit assumption in ANOVA analyses.) To the extent that prior studies cannot be cast in models that are linear in parameters, the approach cannot be adopted. The methodology developed subsequently, however, is not confined to cases in which disturbances are characterized by scalar variance–covariance matrices as defined in (2). We confined ourselves to this case here for simplicity.

The fundamental assumption of meta-analysis is that systematic variation in the parameter vectors β_i can be related to study characteristics. For example, systematic differences might be attributable to differences in product category, industry, sample size, estimation method, and so forth. These study characteristics define a multidimensional classification within which similar studies can be grouped together. Within each cell, l, of the classification, the random coefficients assumption

$$\beta_j = \bar{\beta}_l + e_j \quad \text{for all } j \in k_l, \quad l = 1, 2, \ldots, k, \tag{3}$$

holds with first and second moments

$$E(e_j) = 0 \quad \text{for all } j \in k_l, \quad l = 1, 2, \ldots, k,$$

and

$$\begin{aligned} E(e_ie_j') &= \Sigma_l \quad \text{if } i = j \text{ and } i, j \in k_l \\ &= 0 \quad \text{otherwise.} \end{aligned}$$

In these expressions, k_l denotes the set of studies in group l of k groups. In hierarchical Bayes prior (e.g., see Kuczera 1983), the parameter vectors β_i for *all i* would be assumed to be drawn from a prior distribution (most likely normal) with a mean vector and covariance matrix. Accordingly, the β_i's are exchangeable, and this prior distribution captures the "connectedness" between all studies (DuMouchel and Harris 1983). In other words, each parameter vector of a previous study contributes information about any single parameter in the basic linear model. This strict assumption can be tested (e.g., see Kuczera 1983) and could lead to a grouping of studies that could be analyzed *separately*. (Prior grouping is, of course, also possible, but general lack of prior information on group membership makes this impractical.) Capturing and describing the heterogeneity explicitly as achieved in meta-analysis enables *simultaneous* analysis, which enhances efficiency and understanding.

According to the study design (and hence mean parameter vector), the m studies can be grouped together and their parameter vectors stacked on top of one another to form the equation $\beta = \bar{\beta} + e$, where $\beta' = (\beta_1', \beta_2', \ldots, \beta_m')$, $\bar{\beta}' = (\bar{\beta}_1', \ldots, \bar{\beta}_k')$, and $e' = (e_1', e_2', \ldots, e_m')$. Note that vector $\bar{\beta}$ has m vector entries, one for each original study; the entries corresponding to studies grouped together are identical $\bar{\beta}_i$ vectors. In other words, if the first three studies belong to group 1, then $\bar{\beta}' = (\bar{\beta}_1', \bar{\beta}_1', \bar{\beta}_1', \bar{\beta}_2', \ldots)$.

Of specific interest in meta-analysis are the variations in the mean vectors $\bar{\beta}_l$ when $l = 1, 2, \ldots, k$. Systematic variations arising from design characteristics imply that $\bar{\beta} = Zc$ and hence

$$\beta = Zc + e; \tag{4}$$

that is, the mean parameter vectors can be expressed as a linear combination, c, of specific design vectors that constitute the columns of matrix Z. According to their profile on the design variables, the m original studies are classified into k groups. These variables (commonly called moderator variables) could be continuous, categorical, or dummy variables. Note that Expression (4) not only describes the heterogeneity among the previous studies but at the same time captures the systematic link among them.

In practice, the parameter vectors are not observed. Traditional meta-analysis relies on parameter *estimates* in attempting to estimate vector c. Hence the basic model postulated is

$$\hat{\beta} = Zc + \varepsilon, \tag{5}$$

where $\hat{\beta}' = (\hat{\beta}_1', \hat{\beta}_2', \ldots, \hat{\beta}_m')$, with $\hat{\beta}_i$ containing unbiased least squares estimates of the parameters in study i and $\varepsilon = e + \hat{u}$ when $\hat{u} = \hat{\beta} - \beta$ or the difference between the estimated parameters and their true values [i.e., $\hat{u}_i = (X_i'X_i)^{-1}X_i'u_i$ given (2)].

Given the assumptions about the random components in (2), (4), and (5), it is evident that $E(\varepsilon) = 0$ and

$$E(\varepsilon\varepsilon') = \Omega = \begin{bmatrix} \Sigma_1 + \sigma_1^2(X_1'X_1)^{-1} \ldots 0 \\ \vdots \\ 0 \ldots \Sigma_k + \sigma_m^2(X_m'X_m)^{-1} \end{bmatrix}. \tag{6}$$

Invoking GLS principles given the nonscalar variance–covariance matrix in (6), best linear unbiased estimator (BLUE) estimates of vector c in (4) are

$$\hat{c} = (Z'\Omega^{-1}Z)^{-1}Z'\Omega^{-1}\hat{\beta}. \tag{7}$$

[The result shown in (7) suggests that the most efficient estimator of c in Model (5) is the GLS estimator shown in (7).] Stated simply,

$$\hat{c} = D\hat{\beta}, \qquad D = (Z'\Omega^{-1}Z)^{-1}Z'\Omega^{-1}. \tag{8}$$

In a new study, a regression model similar to the ones specified in (1) can be postulated. Specifically, we have that $y_o = X_o\beta_o + u_o$ when $E(u_o) = 0$ and $E(u_ou_o') = \sigma_o^2 I$. [Whether or not the similarity in model specification is justified depends on the problem studied and is largely determined by theoretical issues. Note, however, that if there is little similarity or analogy between previous studies (incorporated in the meta-analysis) and the new study, the meta-analysis prior will be a noninformative prior.] Given the meta-analysis results in (7), a prior estimate of β_o can be obtained from $b_o = Z_o\hat{c}$, where Z_o defines the design of the current study in terms of the characteristics incorporated in the design matrix Z of the meta-analysis.

As shown in Appendix A, the prior estimate b_o can be expressed as

$$b_o = Z_oDM\beta_o + Z_oDd + Z_oD\gamma, \tag{9}$$

where $\gamma = \xi + \hat{u}$ with D as defined in (8), M being a matrix of m identity matrices stacked on top of one another and d being a vector containing the difference between the mean parameter vector of the corresponding study and the mean parameter vector for the studies belonging to the same set as the new study. This is the meta-analysis prior that will be updated with sample information in the new study using a familiar recursive estimator. The use of this estimator fits nicely into the advocated regression approach and, as will be shown shortly, makes the approach applicable when data in the new study are either incomplete or insufficient for independent estimation.

3. RECURSIVE ESTIMATION

Updating the prior on β_o in (9) with sample information contained in y_o and X_o can be accomplished using the augmented system

$$\begin{bmatrix} y_o \\ b_o - Z_oDd \end{bmatrix} = \begin{bmatrix} X_o \\ Z_oDM \end{bmatrix} \beta_o + \begin{bmatrix} u_o \\ Z_oD\gamma \end{bmatrix}. \tag{10}$$

When the prior information is with respect to a subset of the parameters in β_o (e.g., β_{o1}), the augmented system is

$$\begin{bmatrix} y_o \\ b_{o1} - Z_{o1}D_1d_1 \end{bmatrix} = \begin{bmatrix} X_{o1} \\ Z_{o1}D_1M \end{bmatrix} \beta_{o1} + \begin{bmatrix} X_{o2} \\ 0 \end{bmatrix} \beta_{o2} + \begin{bmatrix} u_o \\ Z_{o1}D_1\gamma_1 \end{bmatrix},$$

where b_{o1} denotes the meta-analysis prior on β_{o1} and all other elements are similar to the corresponding ones in (10) when $\beta_o' = [\beta_{o1}'\beta_{o2}']$ and $X_o = [X_{o1}X_{o2}]$ (where X_{o1} contains the columns in X_o corresponding to the parameters in β_{o1}). The system could be expanded further if independent prior information (e.g., separate meta-analysis results) was available for different subsets of β_o.

This approach is similar to the Goldberger–Theil approach (Theil 1971) with the apparent difference that (a) the prior values for β_o and b_o are adjusted by Z_oDd and (b) the matrix associated with vector β_o in the prior equation [i.e., Z_oDM in (10)] is different from an identity matrix. It is worth noting that $b_o - Z_oDd = Z_o[\hat{c} - Dd]$ and hence $b_o - Z_oDd = Z_o\hat{c} - Z_o(Z'\Omega^{-1}Z)^{-1}Z'\Omega^{-1}d$. Accordingly, Dd can be interpreted as a least squares estimate of the design matrix

Z on the differences in group means. If no groups were identified (i.e., complete exchangeability), the prior would equal $Z_o\hat{c}$. Moreover, Z_oDd is subtracted from this prior to adjust for the partial exchangeability captured in the profiles contained in design matrix Z. In essence, b_o is adjusted in the direction of the mean parameter vector of the group to which the new study belongs.

Expressing (10) compactly, we have

$$y = X\beta_o + v, \tag{11}$$

where $y' = [y_o', (b_o - Z_oDd)']$, $X' = [X_o'(Z_oDM)']$, and $v' = [u_o', (Z_oD\gamma)']$. With the variance–covariance matrix of v, Ω_v as derived in Appendix B, the recursive estimator of β_o is

$$\hat{\beta} = (X'\Omega_v^{-1}X)^{-1}X'\Omega_v^{-1}y, \tag{12}$$

which has BLUE properties.

Some interesting observations can be made about the estimator shown in (12). The development thus far assumed that all information for the new study was available. This is not always the case in practice. For example, data on some predictors contained in X_o could be missing. Hence the data matrix X_o would have columns containing zeros and a direct estimate of β_o could not be obtained. [Replacing missing values with zeros is the traditional approach in data transferability (e.g., see Aigner and Leamer 1984). Other approaches have been suggested in the literature on missing data (e.g., see Dunsmuir and Robinson 1981; Gourieroux and Monfort 1981; Stewart and Sorenson 1981). These approaches rely on likelihood functions and are often impractical. Working with zeros makes possible recursive estimation as a simple, practical, and conceptually appealing method for the problem investigated.] Note, however, that the augmented matrix X in (11) is still of full rank; hence a recursive estimate of all parameters in β_o is obtainable despite the missing data. This is essentially an approach to data transferability (Aigner and Leamer 1984) that, using completely exchangeable priors, was studied by Vanhonacker and Price (1988).

Recursive estimates can also be obtained when the number of observations in y_o and X_o are smaller than the number of predictors specified in the basic model (i.e., the number of columns in X_o). Again, direct least squares estimates of β_o could not be obtained, since $X_o'X_o$ would not be of full rank. The augmented system in (10), however, does provide estimates for β_o in this case, with degrees of freedom equal to the number of observations actually available on the dependent and independent variables.

4. SOME ESTIMATION ISSUES

The preceding discussion assumed that Σ_l $(l = 1, 2, \ldots, k)$, σ_i^2 $(i = 0, 1, 2, \ldots, m)$, and d are known. In practice, these terms will have to be estimated. The term σ_o^2 can be estimated as the average of the variances for the corresponding studies in the meta-analysis

$$\hat{\sigma}_o^2 = (1/m_1)\sum_{i=1}^{m_1}\hat{\sigma}_i^2, \tag{13}$$

where $\hat{\sigma}_i^2 = [(y_i - X_i\hat{\beta}_i)'(y_i - X_i\hat{\beta}_i)/(n-1)]$, $\hat{\beta}_i$ contains the parameter estimates of the ith study, and m_1 denotes the number of previous studies belonging to the first set (to which the new study belongs).

The matrices Σ_l $(l = 1, 2, \ldots, k)$ can be estimated as the average of the covariance matrices for the corresponding studies in the meta-analysis

$$\hat{\Sigma}_i = (1/m_i)\sum_{j=1}^{m_i}(\hat{\beta}_j - \hat{\hat{\beta}}_i)(\hat{\beta}_j - \hat{\hat{\beta}}_i)', \tag{14}$$

where $\hat{\hat{\beta}}_i = \Sigma_{j=1}^{m_i}(\hat{\beta}_j/m_i)$ with m_i denoting the number of studies contained in the ith set. Note that, assuming normality and invoking the Lindley and Smith (1972) results, the estimates in (13) and (14) are related to the empirical Bayes estimates using noninformative priors (see Aigner and Leamer 1984).

Estimates for σ_i^2 $(i = 1, 2, \ldots, m)$ can be obtained from the estimation results of the previous studies. Specifically, the variance–covariance matrices of the parameter estimates from the previous studies are estimates for $\sigma_i^2(X_i'X_i)^{-1}$ $(i = 1, 2, \ldots, m)$ as incorporated in matrix ψ (see App. B). The difference between the mean parameter vectors contained in d can be derived straightforwardly from the averaged estimates $\hat{\hat{\beta}}_l = \Sigma_{j=1}^{m_l}(\hat{\beta}_j/m_l)$ for $l = 1, 2, \ldots, k$.

5. EMPIRICAL ILLUSTRATIONS

Two different empirical illustrations are discussed. The first example relies on two widely cited meta-analyses in the market-response literature. Both focus on a single but different parameter, and they were executed independently. The second example relies on the meta-analysis of both parameters of the linear, discrete analog of a popular model in the diffusion literature.

5.1 Market-Response Models

The market-response literature contains two meta-analyses, one on advertising elasticities (Assmus et al. 1984) and one on price elasticities (Tellis 1988). Accurate estimation of these parameters is mandatory for optimal (e.g., profit-maximizing) advertising spending and pricing decisions. Elasticities are most often estimated using multiplicative model specifications. Given that these can be linearized in parameters, the procedure developed previously can be implemented to enhance parameter efficiency and stability in case of limited samples.

In this illustration, time series observations for a major brand in a low-priced-consumer-product category were used together with the published meta-analyses. Bimonthly observations were available on market share (the response variable), advertising share, and relative price. Relative price was defined as the brand's price over the mean price of the three major competitors in the category. These three competitors together account for over two-thirds of the volume sold in each time period.

A multiplicative response model was specified as

$$\ln \mathrm{MS}_t = \alpha + \beta \ln \mathrm{AS}_t + \lambda \ln \mathrm{RP}_t + \delta \ln \mathrm{MS}_{t-1} + u_t,$$

where MS_t denotes market share at time t, AS_t denotes advertising share at time t, and RP_t denotes relative price at time t. The parameters β and λ denote, respectively, the short-term advertising and price elasticities. Using the meta-analysis prior on β from Assmus et al. (1984) and the meta-analysis prior on λ from Tellis (1988), these parameters were estimated recursively using five, six, seven, and so forth sample observations. Before reporting and discussing the results, a number of practical observations on the implementation of the recursive estimator are in order.

Empirical studies seldom report standard errors of estimated parameters (or exact t values, which would enable derivation of standard errors). The data base underlying the work of Assmus et al. (1984) does not contain any standard errors for the reported advertising elasticities. Tellis's (1988) data base contains some t values but not for all price elasticities. Given that published results are generally statistically significant, the standard errors were derived assuming that the corresponding t values equaled 3. [The importance of incorporating the estimated standard errors was emphasized by Montgomery and Srinivasan (1989).] With this assumption, the entries for the matrices Ω in (6) and ψ in (B.1) of Appendix B could be derived. This assumption affected the weighting of the prior studies incorporated in the meta-analysis data bases as well as the efficiency of the derived parameter estimates.

Although the original data were used (with the original coding schemes), the designs of both meta-analyses were reduced. This was done mainly to ensure that the new study had a design profile that was matched in the data by at least one study. Note that if that study belonged to an empty set, d in (9) cannot be computed and hence no adjustment can be carried out as shown in (10). The design reduction was done based on statistical significance. The Assmus et al. (1984) design matrix Z was reduced from 26 variables to the 11 most significant variables (including the intercept). The Tellis (1988) design matrix Z was reduced from 21 variables to 10 variables (including the intercept). The derived $[b_o - Z_o Dd]$ estimates equaled .0325 for the advertising elasticity and -2.3360 for the price elasticity.

Note that the design profile of the new study matched 2 studies out of 128 in the advertising-elasticity data base but only 1 out of 421 in the price-elasticity data base. This limited representation has a dual effect: First, the information contained in that subgroup will have limited weight in the meta-analysis; second, because the adjustment of b_o towards the subgroup estimate is entirely a function of its members, an inadequate adjustment can arise if these are not representative. These issues will have to be kept in mind when evaluating the results, particularly for the price-response parameter.

The results are summarized in Table 1. Adding a single observation at a time, the table shows the sample ordinary least squares (OLS) estimates (no prior information), the estimates using prior information on one of the two parameters, and the estimates incorporating prior information on both parameters simultaneously. The advertising parameter is very significant in the model, and its estimate gets adjusted very well in the direction of the "long-run" estimate (i.e., the highly significant estimate over the entire sample observation period). For example, the sample estimate with only five observations equals .2310; incorporating prior information, the estimate becomes, respectively, .0727 and .0707, which is much closer to the "long-run" estimate of .0493 and the apparent equilibrium level between .05 and .06. For price, the sample estimate is insignificant. Incorporating the corrected prior value of -2.336 draws the recursive estimate close to that value and makes it insensitive to sample information. As the prior is adjusted toward a single study with a highly significant price elasticity (i.e., a reported t value of -8.33), the significance of the recursive estimate is intuitively unreasonable given limited representation. This raises an issue worth further investigation given the otherwise very encouraging results.

5.2 New-Product Diffusion: The Bass Model

In the modeling literature on the diffusion of innovations over time, the Bass model has received considerable attention (Bass 1969). The basic premise of the model can be stated as

$$P(t) = p + (q/m)Y(t), \tag{15}$$

where $P(t)$ denotes the conditional probability of adoption at time t, $Y(t)$ denotes the cumulative number of adopters by time t, and p, q, and m are the three model parameters. [Parameter p in (15) is labeled the coefficient of *innovation*. Parameter q in (15) is labeled the coefficient of *imitation* because it captures the effect of units sold on the likelihood of adoption. Parameter m in (15) denotes the size of the population that will ultimately adopt the innovation.] Accordingly, the probability of adoption at one point in time is specified as a linear function in the number of adopters by that time. Hence the model describes the adoption process of an innovation as a contagion-type process.

Following the algebra of Bass (1969), it can be shown that in discrete terms the number of adopters in period t, S_t, can be expressed as

$$S_t = a + bY_{t-1} + cY_{t-1}^2, \tag{16}$$

where t refers to a discrete time period (e.g., a month,

Table 1. Elasticity Estimates for Market-Share Response Model[a]

Number of observations	Parameter estimates (estimated variance)[b] based on					
	Sample observations[c]		Meta-analysis prior on single parameter[d]		Meta-analysis prior on both parameters	
	Advertising	Price	Advertising	Price	Advertising	Price
5	.2310(.00)	1.7210(7.48)	.0727(.01)	−2.3355(.00)	.0707(.01)	−2.3356(.01)
6	.1530(.01)	−3.4815(12.8)	.0633(.00)	−2.3363(.00)	.0639(.00)	−2.3359(.01)
7	.1521(.01)	−3.4374(5.39)	.0677(.00)	−2.3365(.00)	.0660(.00)	−2.3362(.00)
8	.1544(.00)	−3.4914(4.67)	.0684(.00)	−2.3365(.00)	.0672(.00)	−2.3363(.00)
9	.1549(.00)	−4.0184(4.70)	.0689(.00)	−2.3367(.00)	.0662(.00)	−2.3365(.00)
10	.1520(.00)	−4.1684(4.05)	.0679(.00)	−2.3368(.00)	.0650(.00)	−2.3366(.00)
11	.0950(.01)	−3.5544(6.43)	.0531(.00)	−2.3366(.00)	.0511(.00)	−2.3364(.00)
12	.0462(.01)	−1.6317(10.7)	.0372(.00)	−2.3357(.01)	.0383(.00)	−2.3356(.01)
13	.0457(.00)	−3.5097(11.0)	.0370(.00)	−2.3367(.01)	.0351(.00)	−2.3367(.01)
14	.0554(.01)	−3.5970(9.87)	.0408(.00)	−2.3367(.01)	.0390(.00)	−2.3367(.01)
15	.0635(.01)	−4.2443(7.81)	.0439(.00)	−2.3373(.01)	.0422(.00)	−2.3373(.01)
16	.0582(.01)	−1.3461(5.05)	.0420(.00)	−2.3348(.01)	.0431(.00)	−2.3348(.01)
17	.0645(.01)	−1.0044(4.21)	.0445(.00)	−2.3342(.01)	.0475(.00)	−2.3341(.01)
18	.0548(6.00)	−1.0553(3.78)	.0452(.00)	−2.3342(.01)	.0558(.00)	−2.3340(.01)
19	.0566(.00)	−1.0573(3.53)	.0475(.00)	−2.3342(.01)	.0592(.00)	−2.3339(.01)
20	.0610(.00)	−1.2296(3.26)	.0504(.00)	−2.3344(.00)	.0605(.00)	−2.3341(.00)
30	.0739(.00)	−.9645(1.45)	.0640(.00)	−2.3320(.00)	.0642(.00)	−2.3320(.00)
40	.0937(.00)	−1.0894(.99)	.0806(.00)	−2.3294(.01)	.0832(.00)	−2.3293(.01)
53	.0493(.00)	.2452(.55)	.0473(.00)	−2.3083(.01)	.0563(.00)	−2.3112(.01)

[a] Multiplicative response model with predictors advertising share, relative price, and lagged market share.

[b] Meta-analysis priors derived from Assmus et al. (1984) and/or Tellis (1988).

[c] Ordinary least squares estimates.

[d] The advertising elasticities shown were estimated using a prior on only that elasticity; the price-elasticity estimates were estimated using a prior on only that elasticity.

quarter, year, etc.) and the parameters a, b, and c are functions of the three original parameters as follows: $a = pm$, $b = q - p$, and $c = (-q/m)$. This quadratic equation was suggested by Bass (1969) as the basic model for empirical analysis.

Since the original article by Bass (1969), substantial research on the specification and estimation of the model has been done. The specification has been extended to incorporate explanatory variables such as price (Jain and Rao 1990), advertising (Horsky and Simon 1983), and so forth. OLS estimation of the parameters of Model (16) has been shown to have a time aggregation bias (Srinivasan and Mason 1986). Maximum likelihood estimation (Schmittlein and Mahajan 1982) as well as nonlinear least squares (NLS) of the continuous-time Bass-model specification have been shown to be preferable over OLS, with NLS being the preferred estimator in terms of fit and predictive validity (Srinivasan and Mason 1986). The linearity-in-parameters constraint of the methodology suggested previously confines us to the OLS estimation of a, b, and c in the discrete analog model (16). Despite the shortcomings of OLS, the estimates are easier to obtain and provide good starting values for NLS. Within these constraints, a number of product/process innovations were analyzed empirically.

A meta-analysis of Bass-diffusion-model estimates is contained in the work of Sultan et al. (in press). That study is very much in the tradition of Assmus et al. (1984) and Tellis (1988), however, and is confined to an independent analysis of individual parameters. To illustrate the full power of the regression framework adopted previously (which allows simultaneous analysis of all parameters), a small meta-analysis was performed on the Bass-model results in Table 2. [Water softeners and boat trailers from, respectively, Bass (1969) and Nevers (1972) were not incorporated because of unrealistic (perhaps misprinted) values.] For simplicity of the illustration and to preserve enough observations for the estimation of the variance–covariance matrices in (14), only a single dummy predictor was specified in (5). In other words, the various innovations listed in Table 2 were broken down into two groups. The variable was whether or not the innovation was a consumer-durable product innovation. A distinction was made between consumer and industrial durables because clear differences exist in their diffusion-process characteristics. The absolute magnitudes of the potential adopter populations are very different, with consumer durables generally having much larger population figures than nonconsumer durables. This results in striking differences in the values of, for example, the parameter a in Model (16), which captures the number of adopters in the initial period of market introduction.

As indicated in the derivation of Expression (7), the covariances of the estimated parameters should be incorporated to secure efficient estimation of the meta-analysis parameters [i.e., vector c in (5)]. These covariances are not reported in the studies whose results are shown in Table 2, however. Only the variances or standard deviations of the parameter estimates are given. Accordingly, vector c in Expression (5) was estimated with variance–covariance matrices in which all off-diagonal elements were 0.

The intercept estimates for parameters a, b, and c (representing the coefficients for an industrial product)

Table 2. Bass-Model Regression Results for Consumer/Industrial Innovation

Product/technology	*Time period*	*Number of observations*	$\hat{a}$	$\hat{b}$	$\hat{c}$	R^2
Electric refrigerators	1920–1940	21	104.67(10^3)	.21305	$-$.053913(10^{-7})	.903
Home freezers	1946–1961	16	308.12(10^3)	.15298	$-$.077868(10^{-7})	.742
Black-and-white TV's	1946–1961	16	2,696.20(10^3)	.23317	$-$.025957(10^{-7})	.576
Room air conditioners	1946–1961	16	175.69(10^3)	.40820	$-$.247770(10^{-7})	.911
Clothes dryers	1948–1961	14	259.67(10^3)	.33968	$-$.236470(10^{-7})	.896
Power lawn mowers	1948–1961	14	410.98(10^3)	.32871	$-$.075506(10^{-7})	.932
Electric bed coverings	1949–1961	13	450.04(10^3)	.23800		.976
Automatic coffee makers	1948–1961	14	1,008.20(10^3)	.28435	$-$.051242(10^{-7})	.883
Steam irons	1949–1960	12	1,594.70(10^3)	.29928	$-$.058875(10^{-7})	.828
Record players	1952–1961	10	543.94(10^3)	.62931	$-$.298170(10^{-7})	.899
Holiday Inn motels	1954–1968	15	9.27866	.35521	$-$.000258	.918
Howard Johnson motor lodges	1956–1969	11	10.64810	.23687	$-$.000502	.759
Ramada Inns	1962–1969	8	14,29150	.14904	.004850	.746
Howard Johnson, Holiday Inn, Ramada Inns	1956–1969	11	24.58870	.32699	$-$.000150	.930
McDonald's restaurants	1955–1965	11	15.00440	.52016	$-$.000646	.964
Rapid bleach process	1959–1968	10	1.86071	.54948	$-$.014050	.919
Conversion to 70% H_2O_2 delivery system	1962–1968	7	5.26971	1.02542	$-$.006090	.974
Continuous bleach ranges	1944–1951	8	2.63906	.90471	$-$.014600	.827
Hybrid corn	1933–1941	9	3.51145	.96654	$-$.011107	.926

NOTE: Results are from Bass (1969) and Nevers (1972).

were equal to, respectively, 8.52905, .56810, and $-.0049512$; the additive effect for a consumer-durable innovation equaled, respectively, 641,196.0, $-.25537$, and .0049511. These additive effects essentially indicate that (as discussed previously) the potential adopter populations for consumer durables are much larger than those for industrial innovations and consumer-durable innovations diffuse slower on their own than industrial innovations. The R^2 equaled .36. Given cross-sectional observations and a single predictor, this fit is impressive.

The methodology incorporating the meta-analysis results discussed previously was applied to forecasting sales of color televisions. The sales data were obtained from Wind and Mahajan (1985, p. 215) and consist of annual observations from 1963 until 1970. Some observations are in order with respect to the product and the data. The Bass model has been fitted quite successfully to the color-television data (Bass 1969; Nevers 1972). Predictions of peak sales (which occur within the period of observation) and the magnitude of the peak have been quite accurate. Comparison of the results derived here with the published independent Bass-model predictions will constitute a relatively strong test of the power of the recursive estimator.

The recursive estimation results and their implied estimates for the parameters of Model (15) are shown in Table 3. [Since $\hat{\sigma}_i^2$ (i = 1, 2, . . . , m) were not available for previous studies, $\hat{\sigma}_o^2$ as shown in (13) could not be derived. The value for $\hat{\sigma}_o^2$ incorporated in the variance–covariance matrix Ω_v was set equal to the unit-sales figure in year 1.] Three sets of results are shown—one set of prior estimates derived from the meta-analysis estimates, one set derived from the priors *and* the first annual sales observations for color televisions, and one set derived from the priors *and* the first two annual sales observations. Estimation results with more data points converged to the least squares estimates based exclusively on color-television sales observations given by Bass (1969) and Nevers (1972). This rapid convergence indicates that the sample observations contain much more information than the prior information. This is intuitively reasonable, since color televisions, relative to the innovations listed in Table 2, are more an improvement of an existing product (i.e., black-and-white television) than a discontinuous innovation. Accordingly, one might expect the faster rate of diffusion observed in the recursive estimates in Table 3 as sample observations are added.

Table 3. Estimation Results for Color Televisions

		Recursive estimation	
Parameter	*Meta-analysis prior*	*One observation incorporated*	*Two observations incorporated*
$\hat{a}(10^3)$	641.205	747.000	747.000
$\hat{b}$	.31273	.41389	1.01309
$\hat{c}(10^{-7})$	$-.10810$	$-.13918$	$-.42612$
$m(10^6)$	30.852	31.445	24.489
p	.021	.024	.031
q	.334	.438	1.044
T^{*a}	7.834	6.315	3.289
$S(T^*)(10^6)^b$	2.903	3.824	6.579

[a] Time of peak sales, or $T^* = [1/(p + q)7[\ln(q/p)]$ (Bass 1969).

[b] Magnitude of peak sales, or $S(T^*) = [m(p + q)^2]/4q$ (Bass 1969).

The corresponding unit sales forecasts are shown in Table 4. The pattern of rapid diffusion is evident when comparing actual sales to the predictions based on the prior estimates. Adding the first-year sales figure adjusts the forecasts in the right direction, but there is still a significant underprediction of actual sales. When the second year is added, the pattern is adjusted further. A reasonable forecast is obtained with a slight overprediction in the post-sample years. This early 1965 forecast predicts sales to peak in 1967 below the 7-

Table 4. Forecasts for Color Televisions

		Forecast based on		
			Recursive estimation	
	Actual	Meta-analysis prior	One observation incorporated	Two observations incorporated
Peak sales*				
1968	5.982			
1972		2.903		
1970			3.824	
1967				6.579
Sales*				
1963	.747	.641	.747	.747
1964	1.480	.837	1.048	1.480
1965	2.646	1.077	1.445	2.792
1966	5.118	1.370	1.942	4.758
1967	5.777	1.702	2.518	6.579
1968	5.982	2.059	3.109	5.915

* Magnitude (10^6).

million-unit point. This forecast is more accurate and much more reasonable than the industry expectations at that time (and even later; see Bass 1969, p. 224). [With more than two sample observations, the results (not shown) converge to the accurate forecasts shown by Bass (1969) and Nevers (1972).] With a better-fitting meta-analysis model than the one relied on in this illustration, the early forecast is likely to improve further.

6. SUMMARY AND CONCLUSION

Meta-analysis has been postulated in a variety of fields as a way to integrate research findings across studies. Within an ANOVA framework, the parameter estimates of the various studies are viewed as imperfect replications of one overall but unplanned experiment. This replication framework generalizes and enhances the understanding of systematic variations in research findings.

Meta-analysis results have never been used, however, as a source of prior information in subsequent studies. Within the design underlying the meta-analysis, it is rather straightforward to obtain prior estimates for the parameters of a new study. This article described a methodology to update this prior information with sample information for the new study. The methodology relies on a random-coefficients regression framework and accomplishes the updating task using a recursive estimator. The procedure is versatile and allows for efficient estimation even when independent estimation is infeasible because the data are either incomplete or too sparse. Empirical examples in the market-response-modeling literature and in the diffusion-modeling literature are encouraging.

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APPENDIX A: DERIVATION OF PRIOR ESTIMATE

Given Expression (8), the prior estimate is $b_o = Z_o D \hat{\beta}$, and, since $\hat{\beta} = \beta + \hat{u}$ with terms as specified previously,

$$b_o = Z_o D \beta + Z_o D \hat{u}. \qquad \text{(A.1)}$$

Depending on the design of the new study, it will belong to one of the k groups specified in terms of design characteristics in the meta-analysis. If we arbitrarily assume that the study belongs to the first group (k_1), we can write the parameter vector as $\beta_o = \bar{\beta}_1 + e_o$. Hence

$$\beta_j = \beta_o + e_{jo} \quad \text{for all } j \in k_1, \qquad \text{(A.2)}$$

where $e_{jo} = e_j - e_o$. For any other study l not belonging to set k_1, we have

$$\beta_l = \beta_o + (\bar{\beta}_i - \bar{\beta}_1) + e_{lo}, \qquad \text{(A.3)}$$

with $\bar{\beta}_i$ denoting the mean parameter vector for the set of studies containing study l.

Combining Expressions (A.2) and (A.3) for all studies, we can specify the augmented model $\beta = M\beta_o + d + \xi$, where $\xi = [e'_{1o}, e'_{2o}, \ldots, e'_{mo}]$ and M denotes a matrix of m identity matrices stacked on top of one another. Vector d contains zeros in the entries corresponding to the parameter vectors for all studies belonging to the same set as the new study and the difference in mean parameter vectors (i.e., $\bar{\beta}_i - \bar{\beta}_1$) in the entries corresponding to parameters vectors for all studies that do not belong to the same set as the new study. Accordingly, the prior estimate b_o in (A.1) can be expressed as shown in (9).

APPENDIX B: COMPUTATION OF VARIANCE–COVARIANCE MATRIX Ω_v

Given Expression (11)

$$E(vv') = \Omega_v = \begin{bmatrix} \sigma_o^2 I & 0 \\ 0 & Z_o D E(\gamma\gamma') D' Z_o' \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_o^2 I & 0 \\ 0 & Z_o D \psi D' Z_o' \end{bmatrix},$$

where $E(\gamma\gamma') = \psi$. Accordingly, $\psi = E[(\xi + \hat{u})(\xi + \hat{u})']$.

Assuming that ξ and $\hat{u}$ are independent and knowing that the new study belongs to the first set, it can be shown rather easily that

$$\psi =$$

$$\begin{bmatrix} 2\Sigma_1 + \sigma_1^2(X_1'X_1)^{-1} & \Sigma_1 & \Sigma_1 \ldots \Sigma_1 \\ \Sigma_1 & 2\Sigma_1 + \sigma_2^2 X_2'X_2)^{-1} & \Sigma_1 \ldots \Sigma_1 \\ \Sigma_1 & \Sigma_1 & \ldots \Sigma_1 \\ \cdot & \cdot & \\ \cdot & \cdot & \\ \Sigma_1 & \Sigma_1 & (\Sigma_1 + \Sigma_k) + \sigma_m^2(X_m'X_m)^{-1} \end{bmatrix} \tag{B.1}$$

or simply $\psi = [\Omega + E \otimes \Sigma_1]$, where E denotes an $(m \times m)$ matrix containing all ones and $\otimes$ denotes a Kronecker product. Note that from the expression of the first two diagonal elements in ψ, it is evident that the corresponding prior studies belong to the set that also contains the new study.

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